CHAPTER 9
Topics in Analytic Geometry

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Topics in Analytic Geometry

Section 9.1 Circles and Parabolas

- A **parabola** is the set of all points (x, y) that are equidistant from a fixed line (**directrix**) and a fixed point (**focus**) not on the line.
- The standard equation of a parabola with vertex (h, k) and
  (a) Vertical axis $x = h$ and directrix $y = k - p$ is
  $(x - h)^2 = 4p(y - k),\ p \neq 0.$
  (b) Horizontal axis $y = k$ and directrix $x = h - p$ is
  $(y - k)^2 = 4p(x - h),\ p \neq 0.$
- The tangent line to a parabola at a point P makes **equal angles** with
  (a) the line through P and the focus.
  (b) the axis of the parabola.

Vocabulary Check

1. conic section
2. locus
3. circle, center
4. parabola, directrix, focus
5. vertex
6. axis
7. tangent

1. $x^2 + y^2 = \left(\sqrt{18}\right)^2$

$x^2 + y^2 = 18$

2. $x^2 + y^2 = \left(4\sqrt{2}\right)^2$

$x^2 + y^2 = 32$

3. Radius $= \sqrt{(3 - 1)^2 + (7 - 0)^2}$

$= \sqrt{4 + 49} = \sqrt{53}$

$(x - h)^2 + (y - k)^2 = r^2$

$(x - 3)^2 + (y - 7)^2 = 53$

4. Radius $= \sqrt{[6 - (-2)]^2 + [-3 - 4]^2}$

$= \sqrt{64 + 49} = \sqrt{113}$

$(x - h)^2 + (y - k)^2 = r^2$

$(x - 6)^2 + (y + 3)^2 = 113$

5. Diameter $= 2\sqrt{7} \implies$ radius $= \sqrt{7}$

$(x - h)^2 + (y - k)^2 = r^2$

$(x + 3)^2 + (y + 1)^2 = 7$

6. Diameter $= 4\sqrt{3} \implies$ radius $= 2\sqrt{3}$

$(x - h)^2 + (y - k)^2 = r^2$

$(x - 5)^2 + (y + 6)^2 = 12$

7. $x^2 + y^2 = 49$

Center: $(0, 0)$

Radius: 7

8. $x^2 + y^2 = 1$

Center: $(0, 0)$

Radius: 1

9. $(x + 2)^2 + (y - 7)^2 = 16$

Center: $(-2, 7)$

Radius: 4

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10. $(x+9)^2+(y+1)^2=36$

Center: $(-9,-1)$

Radius: 6

11. $(x-1)^2+y^2=15$

Center: $(1,0)$

Radius: $\sqrt{15}$

12. $x^2+(y+12)^2=24$

Center: $(0,-12)$

Radius: $\sqrt{24}=2\sqrt{6}$

13. $\frac{1}{4}x^2+\frac{1}{4}y^2=1$

$x^2+y^2=4$

Center: $(0,0)$

Radius: 2

14. $\frac{1}{9}x^2+\frac{1}{9}y^2=1$

$x^2+y^2=9$

Center: $(0,0)$

Radius: 3

15. $\frac{4}{3}x^2+\frac{4}{3}y^2=1$

$x^2+y^2=\frac{3}{4}$

Center: $(0,0)$

Radius: $\frac{\sqrt{3}}{2}$

16. $\frac{9}{2}x^2+\frac{9}{2}y^2=1$

$x^2+y^2=\frac{2}{9}$

Center: $(0,0)$

Radius: $\frac{\sqrt{2}}{3}$

17. $(x^2-2x+1)+(y^2+6y+9)=-9+1+9$

$(x-1)^2+(y+3)^2=1$

Center: $(1,-3)$

Radius: 1

18. $(x^2-10x+25)+(y^2-6y+9)=-25+25+9$

$(x-5)^2+(y-3)^2=9$

Center: $(5,3)$

Radius: 3

19. $4\left(x^2+3x+\frac{9}{4}\right)+4(y^2-6y+9)=-41+9+36$

$4\left(x+\frac{3}{2}\right)^2+4(y-3)^2=4$

$\left(x+\frac{3}{2}\right)^2+(y-3)^2=1$

Center: $\left(-\frac{3}{2},3\right)$

Radius: 1

20. $9(x^2+6x+9)+9(y^2-4y+4)=-17+81+36$

$9(x+3)^2+9(y-2)^2=100$

$(x+3)^2+(y-2)^2=\frac{100}{9}$

Center: $(-3,2)$

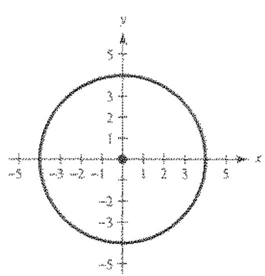
Radius: $\frac{10}{3}$

21. $x^2=16-y^2$

$x^2+y^2=16$

Center: $(0,0)$

Radius: 4

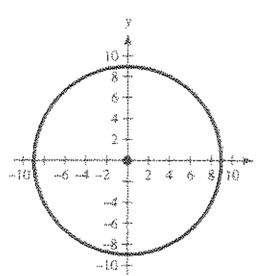


22. $y^2=81-x^2$

$x^2+y^2=81$

Center: $(0,0)$

Radius: 9



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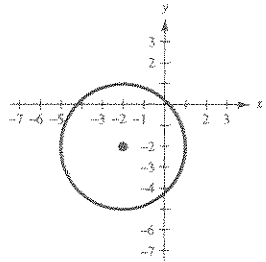
23. $x^2 + 4x + y^2 + 4y - 1 = 0$

$(x^2 + 4x + 4) + (y^2 + 4y + 4) = 1 + 4 + 4$

$(x + 2)^2 + (y + 2)^2 = 9$

Center: $(-2, -2)$

Radius: 3



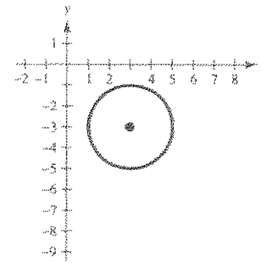
24. $x^2 - 6x + y^2 + 6y + 14 = 0$

$(x^2 - 6x + 9) + (y^2 + 6y + 9) = -14 + 9 + 9$

$(x - 3)^2 + (y + 3)^2 = 4$

Center: $(3, -3)$

Radius: 2



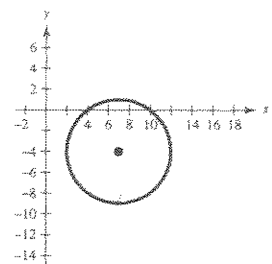
25. $x^2 - 14x + y^2 + 8y + 40 = 0$

$(x^2 - 14x + 4) + (y^2 + 8y + 16) = -40 + 49 + 16$

$(x - 7)^2 + (y + 4)^2 = 25$

Center: $(7, -4)$

Radius: 5



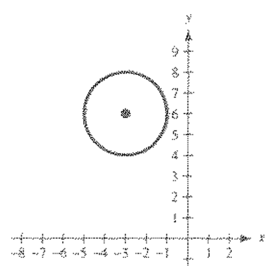
26. $x^2 + 6x + y^2 - 12y + 41 = 0$

$(x^2 + 6x + 9) + (y^2 - 12y + 36) = -41 + 9 + 36$

$(x + 3)^2 + (y - 6)^2 = 4$

Center: $(-3, 6)$

Radius: 2



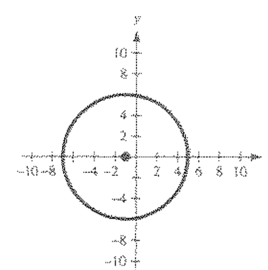
27. $x^2 + 2x + y^2 - 35 = 0$

$(x^2 + 2x + 1) + y^2 = 35 + 1$

$(x + 1)^2 + y^2 = 36$

Center: $(-1, 0)$

Radius: 6



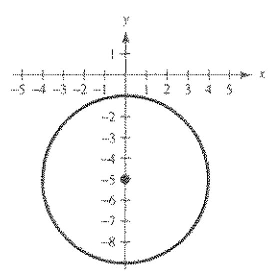
28. $x^2 + y^2 + 10y + 9 = 0$

$x^2 + (y^2 + 10y + 25) = -9 + 25$

$x^2 + (y^2 + 5)^2 = 16$

Center: $(0, -5)$

Radius: 4



29. y-intercepts: $(0 - 2)^2 + (y + 3)^2 = 9$

$4 + (y + 3)^2 = 9$

$(y + 3)^2 = 5$

$y = -3 \pm \sqrt{5}$

$\left(0, -3 \pm \sqrt{5}\right)$

x-intercepts: $(x - 2)^2 + (0 + 3)^2 = 9$

$(x - 2)^2 = 0$

$x = 2$

$(2, 0)$

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30. y-intercepts: $(0 + 5)^2 + (y - 4)^2 = 25$

$$(y-4)^2 = 0$$
$$y = 4$$

$(0, 4)$

x-intercepts: $(x + 5)^2 + (0 - 4)^2 = 25$

$$(x+5)^2 + 16 = 25$$
$$(x+5)^2 = 9$$
$$x + 5 = \pm 3$$
$$x = -8, -2$$

$(-2, 0), (-8, 0)$

31. y-intercepts: Let $x = 0$.

$$y^2 - 6y - 27 = 0$$
$$y^2 - 6y + 9 = 27 + 9$$
$$(y-3)^2 = 36$$
$$y - 3 = \pm 6$$
$$y = 9, -3$$

$(0, 9), (0, -3)$

x-intercepts: Let $y = 0$.

$$x^2 - 2x - 27 = 0$$
$$x^2 - 2x + 1 = 27 + 1$$
$$(x-1)^2 = 28$$
$$x - 1 = \pm\sqrt{28}$$
$$x = 1 \pm 2\sqrt{7}$$

$\left(1 \pm 2\sqrt{7}, 0\right)$

32. y-intercepts: Let $x = 0$.

$$y^2 + 2y + 9 = 0$$

No solution

No y-intercepts

x-intercepts: Let $y = 0$.

$$x^2 + 8x + 9 = 0$$
$$x^2 + 8x + 16 = -9 + 16$$
$$(x+4)^2 = 7$$
$$x + 4 = \pm\sqrt{7}$$
$$x = -4 \pm \sqrt{7}$$

$\left(-4 \pm \sqrt{7}, 0\right)$

33. y-intercepts: $(0 - 6)^2 + (y + 3)^2 = 16$

$$(y+3)^2 = 16 - 36$$
$$= -20$$

No solution

No y-intercepts

x-intercepts: $(x - 6)^2 + (0 + 3)^2 = 16$

$$(x-6)^2 = 7$$
$$x - 6 = \pm\sqrt{7}$$
$$x = 6 \pm \sqrt{7}$$

$\left(6 \pm \sqrt{7}, 0\right)$

34. y-intercepts: $(0 + 7)^2 + (y - 8)^2 = 4$

$$(y-8)^2 = 4 - 49$$
$$= -45$$

No solution

No y-intercepts

x-intercepts: $(x + 7)^2 + (0 - 8)^2 = 4$

$$(x+7)^2 = 4 - 64$$
$$= -60$$

No solution

No x-intercepts

35. (a) Radius: 81; Center: $(0, 0)$

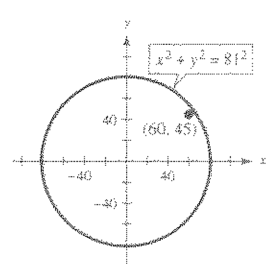
$$x^2 + y^2 = 81^2 = 6561$$

(b) The distance from $(60, 45)$ to $(0, 0)$ is

$\sqrt{60^2 + 45^2} = \sqrt{5625} = 75$ miles.

Yes, you would feel the earthquake.

(c)



You were $81 - 75 = 6$ miles from the outer boundary.

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36. (a) Area $= \pi r^2 = 1800$

$$r^2 = \frac{1800}{\pi}$$

$$r = \sqrt{\frac{1800}{\pi}}$$

$$r \approx 23.937 \text{ feet}$$

(b) $\pi R^2 = 2400$

$$R = \sqrt{\frac{2400}{\pi}} \approx 27.640 \text{ feet}$$

$27.640 - 23.937 \approx 3.703$ longer radius

37. $y^2 = -4x$

Vertex: $(0, 0)$

Opens to the left since p is negative.

Matches graph (e).

38. $x^2 = 2y$

Vertex: $(0, 0)$

$p = \frac{1}{2} > 0$

Opens upward

Matches graph (b).

39. $x^2 = -8y$

Vertex: $(0, 0)$

Opens downward since p is negative.

Matches graph (d).

40. $y^2 = -12x$

Vertex: $(0, 0)$

$p = -3 < 0$

Opens to the left

Matches graph (f).

41. $(y - 1)^2 = 4(x - 3)$

Vertex: $(3, 1)$

Opens to the right since p is positive.

Matches graph (a).

42. $(x + 3)^2 = -2(y - 1)$

Vertex: $(-3, 1)$

$p = -\frac{1}{2} < 0$

Opens downward

Matches graph (c).

43. Vertex: $(0, 0) \implies h = 0, k = 0$

Graph opens upward.

$x^2 = 4py$

Point on graph: $(3, 6)$

$3^2 = 4p(6)$

$9 = 24p$

$\frac{3}{8} = p$

Thus, $x^2 = 4\left(\frac{3}{8}\right)y \implies y = \frac{2}{3}x^2$

$\implies x^2 = \frac{3}{2}y.$

44. Point: $(-2, 6)$

$x = ay^2$

$-2 = a(6)^2$

$-\frac{1}{18} = a$

$x = -\frac{1}{18}y^2$

$y^2 = -18x$

45. Vertex: $(0, 0) \implies h = 0, k = 0$

Focus: $\left(0, -\frac{3}{2}\right) \implies p = -\frac{3}{2}$

$(x - h)^2 = 4p(y - k)$

$x^2 = 4\left(-\frac{3}{2}\right)y$

$x^2 = -6y$

46. Focus: $\left(\frac{5}{2}, 0\right) \implies p = \frac{5}{2}$

$y^2 = 4px = 4\left(\frac{5}{2}\right)x$

$y^2 = 10x$

47. Vertex: $(0, 0) \implies h = 0, k = 0$

Focus: $(-2, 0) \implies p = -2$

$(y - k)^2 = 4p(x - h)$

$y^2 = 4(-2)x$

$y^2 = -8x$

48. Focus: $(0, 1) \implies p = 1$

$x^2 = 4py = 4(1)y$

$x^2 = 4y$

49. Vertex: $(0, 0) \implies h = 0, k = 0$

Directrix: $y = -1 \implies p = 1$

$(x - h)^2 = 4p(y - k)$

$(x - 0)^2 = 4(1)(y - 0)$

$x^2 = 4y$ or $y = \frac{1}{4}x^2$

50. Directrix: $y = 3 \implies p = -3$

$x^2 = 4py$

$x^2 = -12y$

51. Vertex: $(0, 0) \implies h = 0, k = 0$

Directrix: $x = 2 \implies p = -2$

$y^2 = 4px$

$y^2 = -8x$

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52. Directrix: $x = -3 \Rightarrow p = 3$

$y^2 = 4px$

$y^2 = 12x$

53. Vertex: $(0, 0) \Rightarrow h = 0, k = 0$

Horizontal axis and passes through the point $(4, 6)$

$(y - k)^2 = 4p(x - h)$

$(y - 0)^2 = 4p(x - 0)$

$y^2 = 4px$

$6^2 = 4p(4)$

$36 = 16p \Rightarrow p = \frac{9}{4}$

$y^2 = 4\left(\frac{9}{4}\right)x$

$y^2 = 9x$

54. Vertical axis

Passes through $(-3, -3)$

$x^2 = 4py$

$(-3)^2 = 4p(-3)$

$9 = -12p \Rightarrow p = -\frac{3}{4}$

$x^2 = 4\left(-\frac{3}{4}\right)y$

$x^2 = -3y$

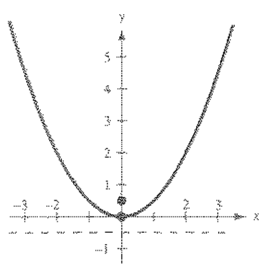
55. $y = \frac{1}{2}x^2$

$x^2 = 2y = 4\left(\frac{1}{2}\right)y;\ p = \frac{1}{2}$

Vertex: $(0, 0)$

Focus: $\left(0, \frac{1}{2}\right)$

Directrix: $y = -\frac{1}{2}$



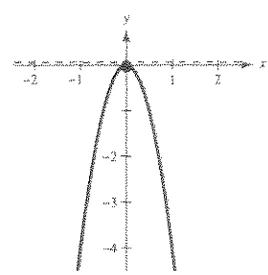
56. $y = -4x^2$

$x^2 = -\frac{1}{4}y = 4\left(-\frac{1}{16}\right)y,\ p = -\frac{1}{16}$

Vertex: $(0, 0)$

Focus: $\left(0, -\frac{1}{16}\right)$

Directrix: $y = \frac{1}{16}$



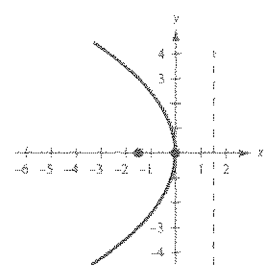
57. $y^2 = -6x$

$y^2 = 4\left(-\frac{3}{2}\right)x;\ p = -\frac{3}{2}$

Vertex: $(0, 0)$

Focus: $\left(-\frac{3}{2}, 0\right)$

Directrix: $x = \frac{3}{2}$



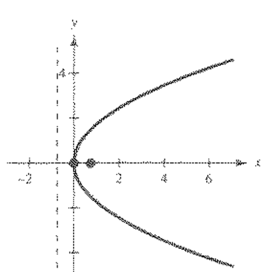
58. $y^2 = 3x$

$y^2 = 4\left(\frac{3}{4}\right)x;\ p = \frac{3}{4}$

Vertex: $(0, 0)$

Focus: $\left(\frac{3}{4}, 0\right)$

Directrix: $x = -\frac{3}{4}$



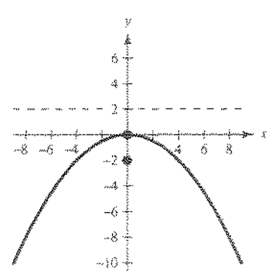
59. $x^2 + 8y = 0$

$x^2 = 4(-2)y;\ p = -2$

Vertex: $(0, 0)$

Focus: $(0, -2)$

Directrix: $y = 2$



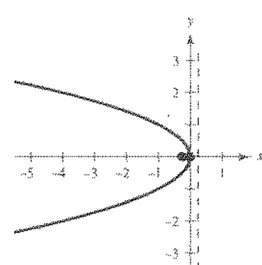
60. $y^2 = -x$

$y^2 = 4\left(-\frac{1}{4}\right)x,\ p = -\frac{1}{4}$

Vertex: $(0, 0)$

Focus: $\left(-\frac{1}{4}, 0\right)$

Directrix: $x = \frac{1}{4}$



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61. $(x + 1)^2 + 8(y + 3) = 0$

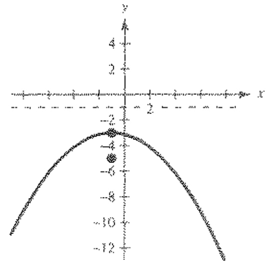
$$(x + 1)^2 = 4(-2)(y + 3)$$

$h = -1, k = -3, p = -2$

Vertex: $(-1, -3)$

Focus: $(-1, -5)$

Directrix: $y = -1$



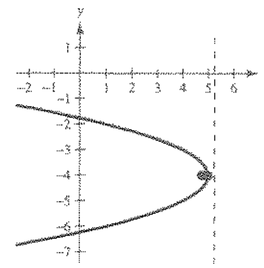
62. $(x - 5) + (y + 4)^2 = 0$

$$(y + 4)^2 = -(x - 5) = 4\left(-\tfrac{1}{4}\right)(x - 5)$$

Vertex: $(5, -4)$

Focus: $\left(5 - \frac{1}{4}, -4\right) = \left(\frac{19}{4}, -4\right)$

Directrix: $x = \frac{21}{4}$



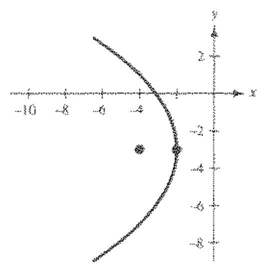
63. $y^2 + 6y + 8x + 25 = 0$

$$(y + 3)^2 = 4(-2)(x + 2); \; p = -2$$

Vertex: $(-2, -3)$

Focus: $(-4, -3)$

Directrix: $x = 0$



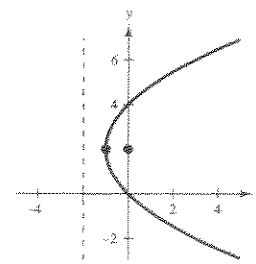
64. $y^2 - 4y - 4x = 0$

$$(y - 2)^2 = 4(x + 1); p = 1$$

Vertex: $(-1, 2)$

Focus: $(0, 2)$

Directrix: $x = -2$

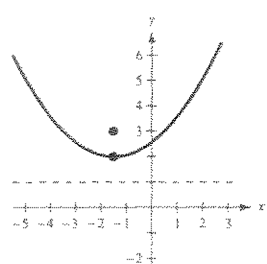


65. $\left(x + \frac{3}{2}\right)^2 = 4(y - 2) \implies h = -\frac{3}{2}, k = 2, p = 1$

Vertex: $\left(-\frac{3}{2}, 2\right)$

Focus: $\left(-\frac{3}{2}, 2 + 1\right) = \left(-\frac{3}{2}, 3\right)$

Directrix: $y = 1$

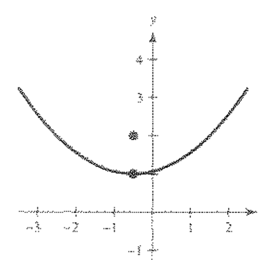


66. $\left(x + \frac{1}{2}\right)^2 = 4(y - 1) \implies p = 1$

Vertex: $\left(-\frac{1}{2}, 1\right)$

Focus: $\left(-\frac{1}{2}, 1 + 1\right) = \left(-\frac{1}{2}, 2\right)$

Directrix: $y = 0$



67. $y = \frac{1}{4}(x^2 - 2x + 5)$

$4y - 4 = (x - 1)^2$

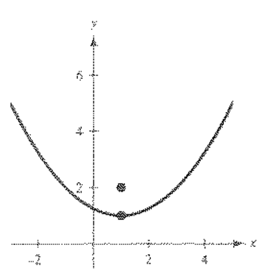
$(x - 1)^2 = 4(1)(y - 1)$

$h = 1, k = 1, p = 1$

Vertex: $(1, 1)$

Focus: $(1, 2)$

Directrix: $y = 0$



68. $4x - y^2 - 2y - 33 = 0$

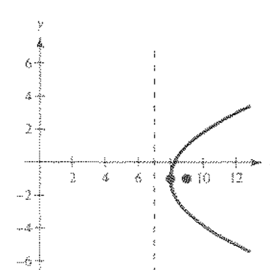
$$y^2 + 2y + 1 = 4x - 33 + 1$$

$$(y + 1)^2 = 4(1)(x - 8)$$

Vertex: $(8, -1)$

Focus: $(9, -1)$

Directrix: $x = 7$



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69. $x^2 + 4x + 6y - 2 = 0$

$$x^2 + 4x + 4 = -6y + 2 + 4 = -6y + 6$$

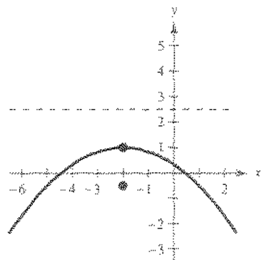
$$(x + 2)^2 = -6(y - 1)$$

$$(x + 2)^2 = 4\left(-\tfrac{3}{2}\right)(y - 1)$$

Vertex: $(-2, 1)$

Focus: $\left(-2, 1 - \frac{3}{2}\right) = \left(-2, -\frac{1}{2}\right)$

Directrix: $y = \frac{5}{2}$



70. $x^2 - 2x + 8y + 9 = 0$

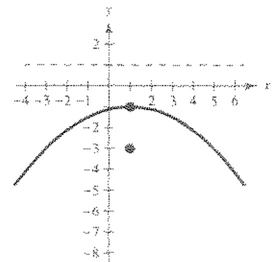
$$x^2 - 2x + 1 = -8y - 9 + 1$$

$$(x - 1)^2 = -8(y + 1) = 4(-2)(y + 1)$$

Vertex: $(1, -1)$

Focus: $(1, -3)$

Directrix: $y = 1$



71. $y^2 + x + y = 0$

$y^2 + y + \frac{1}{4} = -x + \frac{1}{4}$

$\left(y + \frac{1}{2}\right)^2 = 4\left(-\frac{1}{4}\right)\left(x - \frac{1}{4}\right)$

$h = \frac{1}{4}, k = -\frac{1}{2}, p = -\frac{1}{4}$

Vertex: $\left(\frac{1}{4}, -\frac{1}{2}\right)$

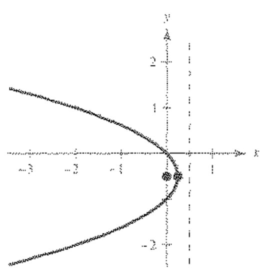
Focus: $\left(0, -\frac{1}{2}\right)$

Directrix: $x = \frac{1}{2}$

To use a graphing calculator, enter:

$y_1 = -\frac{1}{2} + \sqrt{\frac{1}{4} - x}$

$y_2 = -\frac{1}{2} - \sqrt{\frac{1}{4} - x}$



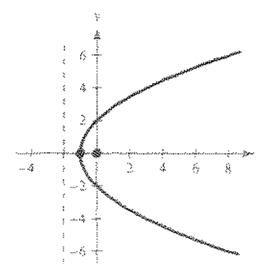
72. $y^2 - 4x - 4 = 0$

$$y^2 = 4x + 4 = 4(1)(x + 1)$$

Vertex: $(-1, 0)$

Focus: $(0, 0)$

Directrix: $x = -2$



73. Vertex: $(3, 1)$, opens downward

Passes through: $(2, 0), (4, 0)$

$$y = -(x - 2)(x - 4)$$
$$= -x^2 + 6x - 8$$
$$= -(x - 3)^2 + 1$$

$(x - 3)^2 = -(y - 1)$

74. Vertex: $(5, 3) \implies h = 5, k = 3$

Passes through: $(4.5, 4)$

$(y - k)^2 = 4p(x - h)$

$(y - 3)^2 = 4p(x - 5)$

$1 = 4p(4.5 - 5)$

$p = -\frac{1}{2}$

$(y - 3)^2 = -2(x - 5)$

75. Vertex: $(-2, 0)$, opens to the right

Focus: $\left(-\frac{3}{2}, 0\right)$

$\frac{1}{2} = p$

$y^2 = 4\left(\frac{1}{2}\right)(x + 2)$

$y^2 = 2(x + 2)$

76. Vertex: $(3, -3) \implies h = 3,\ k = -3$

Focus: $\left(3, -\frac{9}{4}\right) \implies p = \frac{3}{4}$

$(x - h)^2 = 4p(y - k)$

$(x - 3)^2 = 3(y + 3)$

77. Vertex: $(5, 2)$

Focus: $(3, 2)$

Horizontal axis: $p = 3 - 5 = -2$

$(y - 2)^2 = 4(-2)(x - 5)$

$(y - 2)^2 = -8(x - 5)$

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78. Vertex: $(-1, 2) \implies h = -1, k = 2$

Focus: $(-1, 0) \implies p = -2$

$(x - h)^2 = 4p(y - k)$

$(x + 1)^2 = 4(-2)(y - 2)$

$(x + 1)^2 = -8(y - 2)$

79. Vertex: $(0, 4)$

Directrix: $y = 2$

Vertical axis

$p = 4 - 2 = 2$

$(x - 0)^2 = 4(2)(y - 4)$

$x^2 = 8(y - 4)$

80. Vertex: $(-2, 1) \implies h = -2, k = 1$

Directrix: $x = 1 \implies p = -3$

$(y - k)^2 = 4p(x - h)$

$(y - 1)^2 = 4(-3)(x - (-2))$

$(y - 1)^2 = -12(x + 2)$

81. Focus: $(2, 2)$

Directrix: $x = -2$

Horizontal axis

Vertex: $(0, 2)$

$p = 2 - 0 = 2$

$(y - 2)^2 = 4(2)(x - 0)$

$(y - 2)^2 = 8x$

82. Focus: $(0, 0)$

Directrix: $y = 4 \implies p = -2$

Vertex: $(0, 2)$

$x^2 = 4(-2)(y - 2)$

$x^2 = -8(y - 2)$

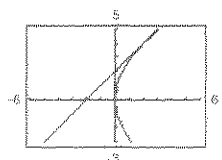
83. $y^2 - 8x = 0$ and $x - y + 2 = 0$

$y^2 = 8x$ $\qquad y_3 = x + 2$

$y_1 = \sqrt{8x}$

$y_2 = -\sqrt{8x}$

The point of tangency is $(2, 4)$.

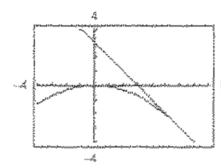


84. $x^2 + 12y = 0$ and $x + y - 3 = 0$

$12y = -x^2$ $\qquad y_2 = 3 - x$

$y_1 = -\frac{1}{12}x^2$

The point of tangency is $(6, -3)$.



85. $x^2 = 2y$, $(4, 8)$, $p = \frac{1}{2}$, focus: $\left(0, \frac{1}{2}\right)$

Following Example 4, we find the y-intercept $(0, b)$.

$$d_1 = \frac{1}{2} - b$$

$$d_2 = \sqrt{(4 - 0)^2 + \left(8 - \frac{1}{2}\right)^2} = \frac{17}{2}$$

$$d_1 = d_2 \implies \frac{1}{2} - b = \frac{17}{2} \implies b = -8$$

$$m = \frac{8 - (-8)}{4 - 0} = 4$$

$y = 4x - 8$, Tangent line

Let $y = 0 \implies x = 2 \implies x$-intercept $(2, 0)$.

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86. $2y = x^2$

$4\left(\frac{1}{2}\right)y = x^2$

$p = \frac{1}{2}$

Focus: $\left(0, \frac{1}{2}\right)$

$d_1 = \frac{1}{2} - b$

$d_2 = \sqrt{(-3-0)^2 + \left(\frac{9}{2} - \frac{1}{2}\right)^2} = 5$

$\frac{1}{2} - b = 5$

$b = -\frac{9}{2}$

$m = \frac{-(9/2) - (9/2)}{0 + 3} = -3$

Tangent line: $y = -3x - \frac{9}{2} \implies 6x + 2y + 9 = 0$

x-intercept: $\left(-\frac{3}{2}, 0\right)$

87. $y = -2x^2 \implies x^2 = -\frac{1}{2}y = 4\left(-\frac{1}{8}\right)y$

$\implies p = -\frac{1}{8}$

Focus: $\left(0, -\frac{1}{8}\right)$

Following Example 4, we find the y-intercept $(0, b)$.

$d_1 = \frac{1}{8} + b$

$d_2 = \sqrt{(-1-0)^2 + \left(-2 + \frac{1}{8}\right)^2} = \frac{17}{8}$

$d_1 = d_2 \implies \frac{1}{8} + b = \frac{17}{8} \implies b = 2$

$m = \frac{-2-2}{-1-0} = 4$

$y = 4x + 2$

Let $y = 0 \implies x = -\frac{1}{2} \implies$ x-intercept $\left(-\frac{1}{2}, 0\right)$.

88. $y = -2x^2, (2, -8)$

$x^2 = -\frac{1}{2}y = 4\left(-\frac{1}{8}\right)y \implies p = -\frac{1}{8}$

Focus: $\left(0, -\frac{1}{8}\right)$

$d_1 = \frac{1}{8} + b$

$d_2 = \sqrt{(2-0)^2 + \left(-8 + \frac{1}{8}\right)^2} = \frac{65}{8}$

$d_1 = d_2 \implies \frac{1}{8} + b = \frac{65}{8} \implies b = 8$

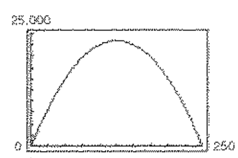
$m = \frac{-8-8}{2-0} = -8$

$y = -8x + 8$

Intercept: $(1, 0)$

89. $R = 375x - \frac{3}{2}x^2$

R is a maximum of \$23,437.50 when $x = 125$ televisions.



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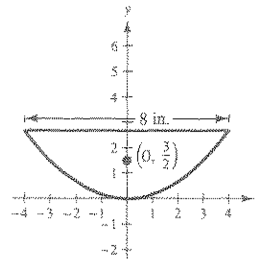
90. (a)
$$x^2 = 4py$$
$$32^2 = 4p\left(\frac{1}{12}\right)$$
$$1024 = \frac{1}{3}p$$
$$3072 = p$$
$$x^2 = 4(3072)y$$
$$y = \frac{x^2}{12{,}288}$$

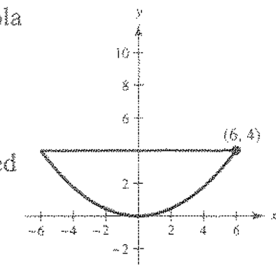
(b)
$$\frac{1}{24} = \frac{x^2}{12{,}288}$$
$$\frac{12{,}288}{24} = x^2$$
$$512 = x^2$$
$$x \approx 22.6 \text{ feet}$$

91. (a) $x^2 = 4py,\ p = \dfrac{3}{2}$
$$x^2 = 4\left(\frac{3}{2}\right)y = 6y$$
(or $y^2 = 6x$)

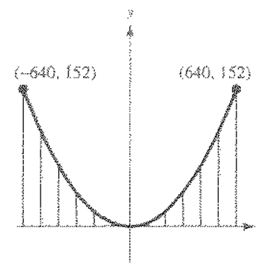
(b) When $x = 4$,
$$6y = 16$$
$$y = \frac{16}{6} = \frac{8}{3}.$$
Depth: $\dfrac{8}{3}$ inches



92. $x^2 = 4py$, $(6, 4)$ on parabola
$$36 = 4p(4)$$
$$p = \tfrac{36}{16} = \tfrac{9}{4}$$
The wire should be inserted $\frac{9}{4}$ inches from the bottom.



93. (a)



(b)
$$x^2 = 4py$$
$$640^2 = 4p(152)$$
$$p = \tfrac{12{,}800}{19}$$
$$y = \tfrac{19}{51{,}200}x^2$$

(c)

x	0	200	400	500	600
y	0	14.84	59.38	92.77	133.59

94. (a) $x^2 = 4py$ passes through point $\left(16, -\frac{2}{5}\right)$.
$$256 = 4p\left(-\tfrac{2}{5}\right) \implies p = -160$$
$$x^2 = 4(-160)y$$
$$x^2 = -640y \text{ or } y = -\tfrac{1}{640}x^2$$
(b) $-0.1 = -\frac{1}{640}x^2 \implies x = 8$ feet

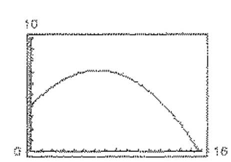
95. Vertex: $(0, 0]$
$$y^2 = 4px$$
Point: $(1000, 800)$
$$800^2 = 4p(1000] \implies p = 160$$
$$y^2 = 4(160)x$$
$$y^2 = 640x$$

96. (a) $V = 17{,}500\sqrt{2}$ mi/hr $\approx 24{,}750$ mi/hr

(b) $p = -4100,\ (h, k) = (0, 4100)$
$$(x - 0)^2 = 4(-4100)(y - 4100)$$
$$x^2 = -16{,}400(y - 4100)$$

97. $-12.5(y - 7.125) = (x - 6.25)^2$
$$-12.5y + 89.0625 = x^2 - 12.5x + 39.0625$$
$$y = -0.08x^2 + x + 4$$

(a)



(b) The highest point is at $(6.25, 7.125)$. The distance is the x-intercept of ≈ 15.69 feet.

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98. (a) $x^2 = -\frac{1}{16}v^2(y - s)$

$$y = -\frac{16x^2}{v^2} + s$$

$$= -\frac{16x^2}{32^2} + 75$$

$$= -\frac{1}{64}x^2 + 75$$

(b) $y = 0 = -\frac{1}{64}x^2 + 75 \implies x^2 = 75(64)$

$$\implies x \approx 69.3 \text{ ft}$$

99. The slope of the line joining $(3, -4)$ and the center is $-\frac{4}{3}$. The slope of the tangent line at $(3, -4)$ is $\frac{3}{4}$. Thus,

$$y + 4 = \frac{3}{4}(x - 3)$$

$$4y + 16 = 3x - 9$$

$$3x - 4y = 25, \quad \text{tangent line.}$$

100. The slope of the line joining $(-5, 12)$ and the center is $-\frac{12}{5}$. The slope of the tangent line at $(-5, 12)$ is $\frac{5}{12}$. Thus,

$$y - 12 = \frac{5}{12}(x + 5)$$

$$12y - 144 = 5x + 25$$

$$5x - 12y + 169 = 0, \quad \text{tangent line.}$$

101. The slope of the line joining $(2, -2\sqrt{2})$ and the center is $(-2\sqrt{2})/2 = -\sqrt{2}$. The slope of the tangent line is $1/\sqrt{2} = \sqrt{2}/2$. Thus,

$$y + 2\sqrt{2} = \frac{\sqrt{2}}{2}(x - 2)$$

$$2y + 4\sqrt{2} = \sqrt{2}x - 2\sqrt{2}$$

$$\sqrt{2}x - 2y = 6\sqrt{2}, \quad \text{tangent line.}$$

102. The slope of the line joining $(-2\sqrt{5}, 2)$ and the center is $2/(-2\sqrt{5}) = -1/\sqrt{5}$. The slope of the tangent line is $\sqrt{5}$. Thus,

$$y - 2 = \sqrt{5}(x + 2\sqrt{5})$$

$$y - 2 = \sqrt{5}x + 10$$

$$\sqrt{5}x - y + 12 = 0, \quad \text{tangent line.}$$

103. False. The center is $(0, -5)$.

104. True

105. False. A circle is a conic section.

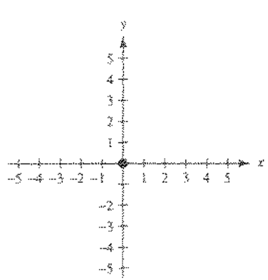
106. False. A parabola cannot intersect its directrix or focus.

107. True

108. False. The directrix $y = -\frac{1}{4}$ is below the x-axis.

109. Answers will vary. See the reflective property of parabolas, page 599.

110. The graph of $x^2 + y^2 = 0$ is a single point, $(0, 0)$.



The plane intersects the double-napped cone at the vertices of the cones.

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111. $(y - 3)^2 = 6(x + 1)$

For the upper half of the parabola,

$y - 3 = \sqrt{6(x + 1)}$

$y = \sqrt{6(x + 1)} + 3.$

112. $(y + 1)^2 = 2(x - 2)$

For the lower half of the parabola,

$y + 1 = -\sqrt{2(x - 2)}$

$y = -1 - \sqrt{2(x - 2)}.$

113. $f(x) = 3x^3 - 4x + 2$

Relative maximum: $(-0.67, 3.78)$

Relative minimum: $(0.67, 0.22)$

114. $f(x) = 2x^2 + 3x$

Relative minimum: -1.13 at $x = -0.75$

115. $f(x) = x^4 + 2x + 2$

Relative minimum: $(-0.79, 0.81)$

116. $f(x) = x^5 - 3x - 1$

Relative minimum: -3.11 at 0.88

Relative maximum: 1.11 at -0.88

Section 9.2 Ellipses

- An **ellipse** is the set of all points (x, y) the sum of whose distances from two distinct fixed points **(foci)** is constant.
- The standard equation of an ellipse with center (h, k) and major and minor axes of lengths $2a$ and $2b$ is
  (a) $\dfrac{(x - h)^2}{a^2} + \dfrac{(y - k)^2}{b^2} = 1$ if the major axis is horizontal.
  (b) $\dfrac{(x - h)^2}{b^2} + \dfrac{(y - k)^2}{a^2} = 1$ if the major axis is vertical.
- $c^2 = a^2 - b^2$ where c is the distance from the center to a focus.
- The eccentricity of an ellipse is $e = \dfrac{c}{a}$.

Vocabulary Check

1. ellipse
2. major axis, center
3. minor axis
4. eccentricity

1. $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$

Center: $(0, 0)$

$a = 3, b = 2$

Vertical major axis

Matches graph (b).

2. $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$

Center: $(0, 0)$

$a = 3, b = 2$

Horizontal major axis

Matches graph (c).

3. $\dfrac{x^2}{4} + \dfrac{y^2}{25} = 1$

Center: $(0, 0)$

$a = 5, b = 2$

Vertical major axis

Matches graph (d).

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4. $\dfrac{x^2}{4} + y^2 = 1$

Center: $(0, 0)$

$a = 2, b = 1$

Horizontal major axis

Matches graph (f).

5. $\dfrac{(x-2)^2}{16} + (y+1)^2 = 1$

Center: $(2, -1)$

$a = 4, b = 1$

Horizontal major axis

Matches graph (a).

6. $\dfrac{(x+2)^2}{9} + \dfrac{(y+2)^2}{4} = 1$

Center: $(-2, -2)$

Horizontal major axis

Matches graph (e).

7. $\dfrac{x^2}{64} + \dfrac{y^2}{9} = 1$

Center: $(0, 0)$

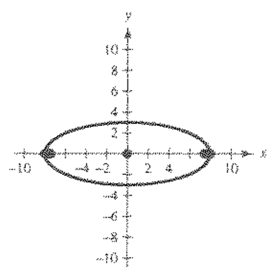
$a = 8, b = 3,$

$c = \sqrt{64 - 9} = \sqrt{55}$

Vertices: $(\pm 8, 0)$

Foci: $\left(\pm\sqrt{55}, 0\right)$

$e = \dfrac{c}{a} = \dfrac{\sqrt{55}}{8}$



8. $\dfrac{x^2}{16} + \dfrac{y^2}{81} = 1$

Center: $(0, 0)$

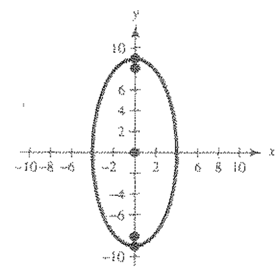
$a = 9, b = 4,$

$c = \sqrt{81 - 16} = \sqrt{65}$

Vertices: $(0, \pm 9)$

Foci: $\left(0, \pm\sqrt{65}\right)$

$e = \dfrac{c}{a} = \dfrac{\sqrt{65}}{9}$



9. $\dfrac{(x-4)^2}{16} + \dfrac{(y+1)^2}{25} = 1$

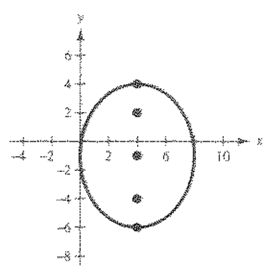
Center: $(4, -1)$

$a = 5, b = 4, c = 3$

Vertices: $(4, -1 \pm 5)$; $(4, -6), (4, 4)$

Foci: $(4, -1 \pm 3)$; $(4, -4), (4, 2)$

$e = \dfrac{c}{a} = \dfrac{3}{5}$



10. $\dfrac{(x+3)^2}{12} + \dfrac{(y-2)^2}{16} = 1$

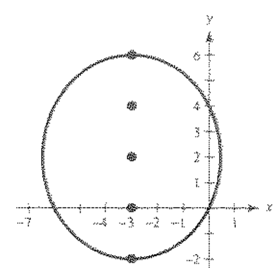
Center: $(-3, 2)$

$a = 4, b = 2\sqrt{3}, c = \sqrt{16 - 12} = 2$

Foci: $(-3, 2 \pm 2)$; $(-3, 0), (-3, 4)$

Vertices: $(-3, 2 \pm 4)$; $(-3, -2), (-3, 6)$

$e = \dfrac{c}{a} = \dfrac{2}{4} = \dfrac{1}{2}$



11. $\dfrac{(x+5)^2}{9/4} + (y-1)^2 = 1$

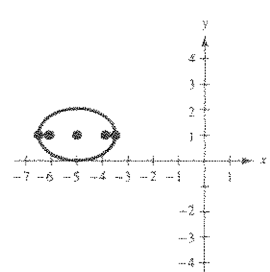
Center: $(-5, 1)$

$a = \dfrac{3}{2}, b = 1, c = \sqrt{\dfrac{9}{4} - 1} = \dfrac{\sqrt{5}}{2}$

Foci: $\left(-5 + \dfrac{\sqrt{5}}{2}, 1\right), \left(-5 - \dfrac{\sqrt{5}}{2}, 1\right)$

Vertices: $\left(-5 + \dfrac{3}{2}, 1\right) = \left(-\dfrac{7}{2}, 1\right), \left(-5 - \dfrac{3}{2}, 1\right) = \left(-\dfrac{13}{2}, 1\right)$

$e = \dfrac{\sqrt{5}/2}{3/2} = \dfrac{\sqrt{5}}{3}$



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12. $(x + 2)^2 + \dfrac{(y + 4)^2}{1/4} = 1$

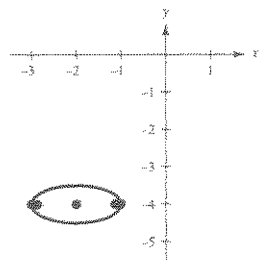
$a = 1, b = \dfrac{1}{2}, c = \sqrt{a^2 - b^2} = \dfrac{\sqrt{3}}{2}$

Center: $(-2, -4)$

Foci: $\left(-2 + \dfrac{\sqrt{3}}{2}, -4\right), \left(-2 - \dfrac{\sqrt{3}}{2}, -4\right)$

Vertices: $(-3, -4), (-1, -4)$

Eccentricity: $\dfrac{\sqrt{3}}{2}$



13. (a) $x^2 + 9y^2 = 36$

$\dfrac{x^2}{36} + \dfrac{y^2}{4} = 1$

(b) $a = 6, b = 2, c = \sqrt{36 - 4} = \sqrt{32} = 4\sqrt{2}$

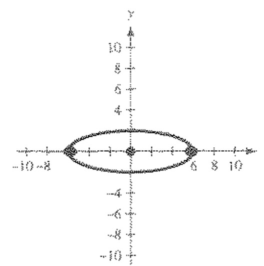
Center: $(0, 0)$

Vertices: $(\pm 6, 0)$

Foci: $(\pm 4\sqrt{2}, 0)$

$e = \dfrac{c}{a} = \dfrac{4\sqrt{2}}{6} = \dfrac{2\sqrt{2}}{3}$

(c)



14. (a) $16x^2 + y^2 = 16$

$x^2 + \dfrac{y^2}{16} = 1$

(b) $a = 4, b = 1, c = \sqrt{16 - 1} = \sqrt{15}$

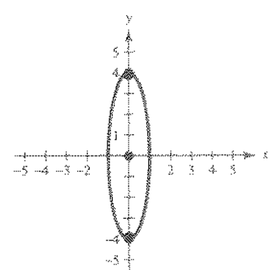
Center: $(0, 0)$

Vertices: $(0, \pm 4)$

Foci: $(0, \pm\sqrt{15})$

$e = \dfrac{c}{a} = \dfrac{\sqrt{15}}{4}$

(c)



15. (a) $9x^2 + 4y^2 + 36x - 24y + 36 = 0$

$9(x^2 + 4x + 4) + 4(y^2 - 6y + 9) = -36 + 36 + 36$

$\dfrac{(x + 2)^2}{4} + \dfrac{(y - 3)^2}{9} = 1$

(b) $a = 3, b = 2, c = \sqrt{5}$

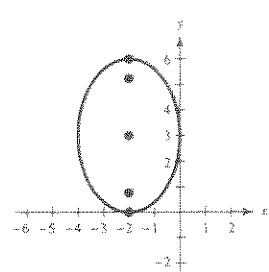
Center: $(-2, 3)$

Foci: $(-2, 3 \pm \sqrt{5})$

Vertices: $(-2, 6), (-2, 0)$

$e = \dfrac{\sqrt{5}}{3}$

(c)



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16. (a) $9(x^2 - 6x + 9) + 4(y^2 + 10y + 25) = -37 + 81 + 100$

$$9(x-3)^2 + 4(y+5)^2 = 144$$

$$\frac{(x-3)^2}{16} + \frac{(y+5)^2}{36} = 1$$

(b) $a = 6, b = 4, c = \sqrt{20} = 2\sqrt{5}$

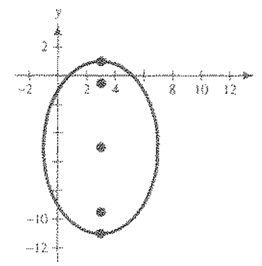
Center: $(3, -5)$

Foci: $\left(3, -5 \pm 2\sqrt{5}\right)$

Vertices: $(3, -5 \pm 6); (3, 1), (3, -11)$

$$e = \frac{2\sqrt{5}}{6} = \frac{\sqrt{5}}{3}$$

(c)



17. (a)

$$6x^2 + 2y^2 + 18x - 10y + 2 = 0$$

$$6\left(x^2 + 3x + \frac{9}{4}\right) + 2\left(y^2 - 5y + \frac{25}{4}\right) = -2 + \frac{27}{2} + \frac{25}{2}$$

$$6\left(x + \frac{3}{2}\right)^2 + 2\left(y - \frac{5}{2}\right)^2 = 24$$

$$\frac{\left(x + \frac{3}{2}\right)^2}{4} + \frac{\left(y - \frac{5}{2}\right)^2}{12} = 1$$

(b) $a = 2\sqrt{3}, b = 2, c = 2\sqrt{2}$

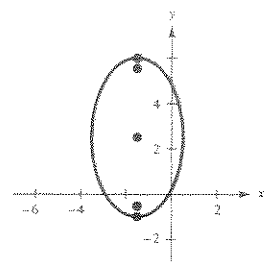
Center: $\left(-\frac{3}{2}, \frac{5}{2}\right)$

Foci: $\left(-\frac{3}{2}, \frac{5}{2} \pm 2\sqrt{2}\right)$

Vertices: $\left(-\frac{3}{2}, \frac{5}{2} \pm 2\sqrt{3}\right)$

$$e = \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$$

(c)



18. (a) $(x^2 - 6x + 9) + 4\left(y^2 + 5y + \frac{25}{4}\right) = 2 + 9 + 25$

$$(x-3)^2 + 4\left(y + \frac{5}{2}\right)^2 = 36$$

$$\frac{(x-3)^2}{36} + \frac{\left(y + \frac{5}{2}\right)^2}{9} = 1$$

(b) $a = 6, b = 3, c = \sqrt{36 - 9} = \sqrt{27} = 3\sqrt{3}$

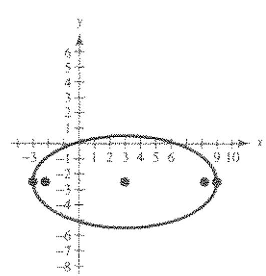
Center: $\left(3, -\frac{5}{2}\right)$

Foci: $\left(3 \pm 3\sqrt{3}, -\frac{5}{2}\right)$

Vertices: $\left(9, -\frac{5}{2}\right), \left(-3, -\frac{5}{2}\right)$

$$e = \frac{\sqrt{3}}{2}$$

(c)



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19. (a) $16x^2 + 25y^2 - 32x + 50y + 16 = 0$

$16(x^2 - 2x + 1) + 25(y^2 + 2y + 1) = -16 + 16 + 25$

$\dfrac{(x-1)^2}{25/16} + (y+1)^2 = 1$

(b) $a = \dfrac{5}{4}, b = 1, c = \dfrac{3}{4}$

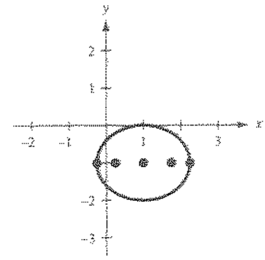
Center: $(1, -1)$

Foci: $\left(\dfrac{7}{4}, -1\right), \left(\dfrac{1}{4}, -1\right)$

Vertices: $\left(\dfrac{9}{4}, -1\right), \left(-\dfrac{1}{4}, -1\right)$

$e = \dfrac{3}{5}$

(c)



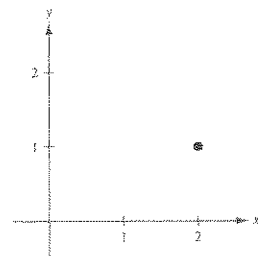
20. (a) $9x^2 + 25y^2 - 36x - 50y + 61 = 0$

$9(x^2 - 4x + 4) + 25(y^2 - 2y + 1) = -61 + 36 + 25$

$9(x-2)^2 + 25(y-1)^2 = 0$

(b) Degenerate ellipse with center $(2, 1)$ as the only point

(c)



21. (a) $12x^2 + 20y^2 - 12x + 40y - 37 = 0$

$12\left(x^2 - 1 + \dfrac{1}{4}\right) + 20(y^2 + 2y + 1) = 37 + 3 + 20$

$12\left(x - \dfrac{1}{2}\right)^2 + 20(y+1)^2 = 60$

$\dfrac{(x - \frac{1}{2})^2}{5} + \dfrac{(y+1)^2}{3} = 1$

(b) $a = \sqrt{5}, b = \sqrt{3}, c = \sqrt{5-3} = \sqrt{2}$

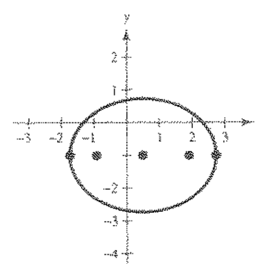
Center: $\left(\dfrac{1}{2}, -1\right)$

Vertices: $\left(\dfrac{1}{2} \pm \sqrt{5}, -1\right)$

Foci: $\left(\dfrac{1}{2} \pm \sqrt{2}, -1\right)$

Eccentricity: $\dfrac{c}{a} = \dfrac{\sqrt{2}}{\sqrt{5}} = \dfrac{\sqrt{10}}{5}$

(c)



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22. (a)
$$36x^2 + 9y^2 + 48x - 36y + 43 = 0$$
$$36\left(x^2 + \frac{4}{3}x + \frac{4}{9}\right) + 9(y^2 - 4y + 4) = -43 + 16 + 36$$
$$36\left(x + \frac{2}{3}\right)^2 + 9(y - 2)^2 = 9$$
$$\frac{\left(x + \frac{2}{3}\right)^2}{\frac{1}{4}} + \frac{(y - 2)^2}{1} = 1$$

(b) $a = 1,\ b = \frac{1}{2},\ c = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}$

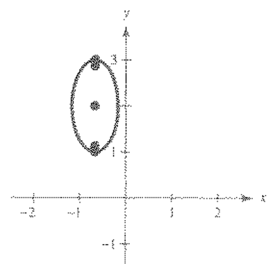
Center: $\left(-\frac{2}{3}, 2\right)$

Vertices: $\left(-\frac{2}{3}, 2 \pm 1\right) = \left(-\frac{2}{3}, 1\right), \left(-\frac{2}{3}, 3\right)$

Foci: $\left(-\frac{2}{3}, 2 \pm \frac{\sqrt{3}}{2}\right)$

Eccentricity: $\frac{c}{a} = \frac{\sqrt{3}}{2}$

(c)



23. Center: $(0, 0)$

$a = 4,\ b = 2$

Vertical major axis

$$\frac{x^2}{4} + \frac{y^2}{16} = 1$$

24. Vertices: $(\pm 2, 0) \implies a = 2$

Endpoints of minor axis: $\left(0, \pm\frac{3}{2}\right) \implies b = \frac{3}{2}$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$
$$\frac{x^2}{2^2} + \frac{y^2}{(3/2)^2} = 1$$
$$\frac{x^2}{4} + \frac{4y^2}{9} = 1$$

25. Center: $(0, 0)$

$a = 3,\ c = 2 \implies b = \sqrt{9 - 4} = \sqrt{5}$

Horizontal major axis

$$\frac{x^2}{9} + \frac{y^2}{5} = 1$$

26. Vertices: $(0, \pm 8) \implies a = 8$

Foci: $(0, \pm 4) \implies c = 4$

$b^2 = a^2 - c^2 = 64 - 16 = 48$

Center: $(0, 0) = (h, k)$

$$\frac{(y - k)^2}{a^2} + \frac{(x - h)^2}{b^2} = 1$$
$$\frac{y^2}{64} + \frac{x^2}{48} = 1$$

27. Center: $(0, 0)$

$c = 3$

$a = 4 \implies b = \sqrt{16 - 9} = \sqrt{7}$

Horizontal major axis

$$\frac{x^2}{16} + \frac{y^2}{7} = 1$$

28. Center: $(0, 0)$

$c = 2$

$a = 6 \implies b = \sqrt{36 - 4} = \sqrt{32} = 4\sqrt{2}$

Horizontal major axis

$$\frac{x^2}{36} + \frac{y^2}{32} = 1$$

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29. Vertices: $(0, \pm 5) \implies a = 5$

Center: $(0, 0)$

Vertical major axis

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

$$\frac{x^2}{b^2} + \frac{y^2}{25} = 1$$

Point: $(4, 2)$

$$\frac{4^2}{b^2} + \frac{2^2}{25} = 1$$

$$\frac{16}{b^2} = 1 - \frac{4}{25} = \frac{21}{25}$$

$$400 = 21b^2$$

$$\frac{400}{21} = b^2$$

$$\frac{x^2}{400/21} + \frac{y^2}{25} = 1$$

$$\frac{21x^2}{400} + \frac{y^2}{25} = 1$$

30. Vertical major axis

Passes through: $(0, 4)$ and $(2, 0)$

$a = 4, b = 2$

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

$$\frac{x^2}{4} + \frac{y^2}{16} = 1$$

31. Center: $(2, 3)$

$a = 3, b = 1$

Vertical major axis

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

$$\frac{(x-2)^2}{1} + \frac{(y-3)^2}{9} = 1$$

32. Vertices: $(0, -1), (4, -1) \implies a = 2$

Center: $(2, -1) \implies h = 2, k = -1$

Endpoints of minor axis: $(2, 0), (2, -2) \implies b = 1$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$\frac{(x-2)^2}{4} + \frac{(y+1)^2}{1} = 1$$

33. Center: $(4, 2)$

$a = 4, b = 1 \implies c = \sqrt{16-1} = \sqrt{15}$

Horizontal major axis

$$\frac{(x-4)^2}{16} + \frac{(y-2)^2}{1} = 1$$

34. Center: $(2, 0)$

$c = 2, a = 3 \implies b^2 = a^2 - c^2 = 9 - 4 = 5$

Horizontal major axis

$$\frac{(x-2)^2}{9} + \frac{y^2}{5} = 1$$

35. Center: $(0, 4)$

$c = 4, a = 18 \implies b^2 = a^2 - c^2 = 324 - 16 = 308$

Vertical major axis

$$\frac{x^2}{308} + \frac{(y-4)^2}{324} = 1$$

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36. Center: $(2, -1) \implies h = 2, k = -1$

Vertex: $\left(2, \frac{1}{2}\right) \implies a = \frac{3}{2}$

Minor axis length: $2 \implies b = 1$

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

$$\frac{(x-2)^2}{1} + \frac{(y+1)^2}{(3/2)^2} = 1$$

$$(x-2)^2 + \frac{4(y+1)^2}{9} = 1$$

37. Vertices: $(3, 1), (3, 9) \implies a = 4$

Center: $(3, 5)$

Minor axis of length 6 $\implies b = 3$

Vertical major axis

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

$$\frac{(x-3)^2}{9} + \frac{(y-5)^2}{16} = 1$$

38. Center: $(3, 2) = (h, k)$

$a = 3c$

Foci: $(1, 2), (5, 2) \implies c = 2, a = 6$

$b^2 = a^2 - c^2 = 36 - 4 = 32$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$\frac{(x-3)^2}{36} + \frac{(y-2)^2}{32} = 1$$

39. Center: $(0, 4)$

Vertices: $(-4, 4), (4, 4) \implies a = 4$

$a = 2c \implies 4 = 2c \implies c = 2$

$2^2 = 4^2 - b^2 \implies b^2 = 12$

Horizontal major axis

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$\frac{x^2}{16} + \frac{(y-4)^2}{12} = 1$$

40. Vertices:
$(5, 0), (5, 12) \implies a = 6$

Endpoints of minor axis:
$(0, 6), (10, 6) \implies b = 5$

Center: $(5, 6) \implies h = 5, k = 6$

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

$$\frac{(x-5)^2}{25} + \frac{(y-6)^2}{36} = 1$$

41. $\frac{x^2}{4} + \frac{y^2}{9} = 1$

$a = 3, b = 2,$

$c = \sqrt{9-4} = \sqrt{5}$

$e = \frac{c}{a} = \frac{\sqrt{5}}{3}$

42. $\frac{x^2}{25} + \frac{y^2}{36} = 1$

$a = 6, b = 5,$

$c = \sqrt{36-25} = \sqrt{11}$

$e = \frac{c}{a} = \frac{\sqrt{11}}{6}$

43.
$$x^2 + 9y^2 - 10x + 36y + 52 = 0$$
$$(x^2 - 10x + 25) + 9(y^2 + 4y + 4) = -52 + 25 + 36$$
$$(x-5)^2 + 9(y+2)^2 = 9$$
$$\frac{(x-5)^2}{9} + \frac{(y+2)^2}{1} = 1$$

$a = 3, b = 1, c = \sqrt{9-1} = 2\sqrt{2}$

$e = \frac{c}{a} = \frac{2\sqrt{2}}{3}$

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44.
$$4x^2 + 3y^2 - 8x + 18y + 19 = 0$$
$$4(x^2 - 2x + 1) + 3(y^2 + 6y + 9) = -19 + 4 + 27$$
$$4(x - 1)^2 + 3(y + 3)^2 = 12$$
$$\frac{(x-1)^2}{3} + \frac{(y+3)^2}{4} = 1$$
$a = 2, b = \sqrt{3}, c = \sqrt{4 - 3} = 1$

$e = \frac{c}{a} = \frac{1}{2}$

45. Vertices: $(\pm 5, 0) \implies a = 5$

Eccentricity: $\frac{4}{5} = \frac{c}{a} \implies c = \frac{4}{5}a = 4$

$b^2 = a^2 - c^2 = 25 - 16 = 9$

Center: $(0, 0)$

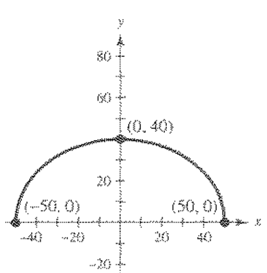
Horizontal major axis

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

46. Vertices: $(0, \pm 8) \implies a = 8, h = 0, k = 0$

Eccentricity: $e = \frac{1}{2} = \frac{c}{a}$
$$\frac{1}{2} = \frac{c}{8}$$
$$c = 4$$
$b^2 = a^2 - c^2 = 64 - 16 = 48$
$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$
$$\frac{x^2}{48} + \frac{y^2}{64} = 1$$

47. (a)



(b) Vertices: $(\pm 50, 0) \implies a = 50$

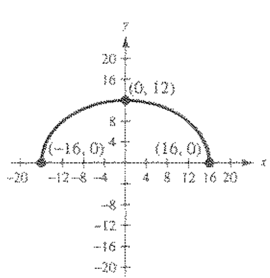
Height at center: $40 \implies b = 40$

Horizontal major axis

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$
$$\frac{x^2}{2500} + \frac{y^2}{1600} = 1,\ y \ge 0$$

(c) For $x = 45$, $\frac{45^2}{2500} + \frac{y^2}{1600} = 1$.
$$y^2 = 1600\left(1 - \frac{45^2}{2500}\right)$$
$$y^2 = 304$$
$$y \approx 17.44$$
The height five feet from the edge of the tunnel is approximately 17.44 feet.

48. (a)



(b) $a = 16, b = 12$
$$\frac{x^2}{256} + \frac{y^2}{144} = 1,\ y \ge 0$$

(c) When $x = 10$,
$$y^2 = 144\sqrt{1 - \frac{10^2}{256}}$$
$$y \approx 9.4 > 9.$$
Hence, the truck will be able to drive through without crossing the center line.

49. Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ be the equation of the ellipse. Then $b = 2$ and $a = 3 \implies c^2 = a^2 - b^2 = 9 - 4 = 5$. Thus, the tacks are placed at $(\pm\sqrt{5}, 0)$. The string has a length of $2a = 6$ feet.

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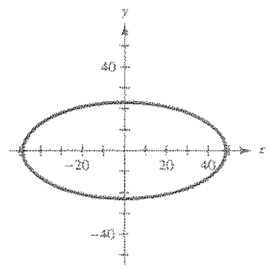
50.



$$\frac{x^2}{(97/2)^2} + \frac{y^2}{23^2} = 1 \quad \left(\text{or } \frac{x^2}{23^2} + \frac{y^2}{(97/2)^2} = 1\right)$$

$$a = \frac{97}{2}, b = 23, c = \sqrt{\left(\frac{97}{2}\right)^2 - (23)^2} \approx 4.7$$

Distance between foci: $2(4.7) \approx 85.4$ feet

51. Area of ellipse $=$ 2(area of circle)

$$\pi ab = 2\pi r^2$$
$$\pi a(10) = 2\pi(10)^2$$
$$\pi a(10) = 200$$
$$a = 20$$

Length of major axis: $2a = 2(20) = 40$ units

52. Center: $(0, 0)$, $e = 0.97$

$2a = 35.88 \implies a = 17.94 \implies a^2 \approx 321.84$

$e = \dfrac{c}{a} \implies 0.97 = \dfrac{c}{17.94} \implies c = 17.4018$

$c^2 = a^2 - b^2 \implies b^2 = a^2 - c^2 \approx 19.02$

Ellipse: $\dfrac{x^2}{321.84} + \dfrac{y^2}{19.02} = 1$

53. $a + c = 4.08$

$a - c = 0.34$

$2a = 4.42 \implies a = 2.21 \implies c = 1.87$

$b^2 = a^2 - c^2 \implies b^2 = 1.3872$

$\dfrac{x^2}{4.8841} + \dfrac{y^2}{1.3872} = 1$

54. $a + c = 947 + 6378 = 7325$

$a - c = 228 + 6378 = 6606$

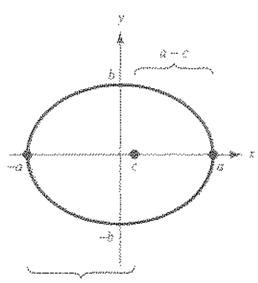
$2a = 13{,}931$

$a = 6965.5$

$c = 7325 - 6965.5$

$= 359.5$

$e = \dfrac{c}{a} \approx 0.0516$



55. For $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, we have $c^2 = a^2 - b^2$.

When $x = c$,

$$\frac{c^2}{a^2} + \frac{y^2}{b^2} = 1 \implies y^2 = b^2\left(1 - \frac{a^2 - b^2}{a^2}\right)$$
$$\implies y^2 = \frac{b^4}{a^2}$$
$$\implies 2y = \frac{2b^2}{a}.$$

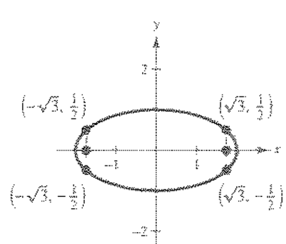
56. $\dfrac{x^2}{4} + \dfrac{y^2}{1} = 1$

$a = 2,\ b = 1,\ c = \sqrt{3}$

Points on the ellipse:
$(\pm 2, 0), (0, \pm 1)$

Length of latus recta:

$\dfrac{2b^2}{a} = 1$



Additional points: $\left(\sqrt{3}, \pm\dfrac{1}{2}\right), \left(-\sqrt{3}, \pm\dfrac{1}{2}\right)$

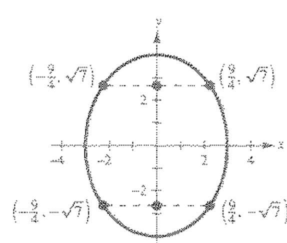
57. $\dfrac{x^2}{9} + \dfrac{y^2}{16} = 1$

$a = 4, b = 3, c = \sqrt{7}$

Points on the ellipse:
$(\pm 3, 0), (0, \pm 4)$

Length of latus recta:

$\dfrac{2b^2}{a} = \dfrac{2(3)^2}{4} = \dfrac{9}{2}$



Additional points: $\left(\pm\dfrac{9}{4}, -\sqrt{7}\right), \left(\pm\dfrac{9}{4}, \sqrt{7}\right)$

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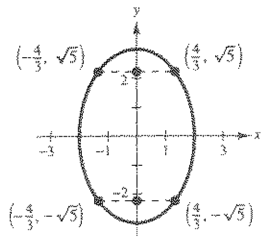
58. $9x^2 + 4y^2 = 36$

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$



Points on the ellipse: $(\pm 2, 0), (0, \pm 3)$

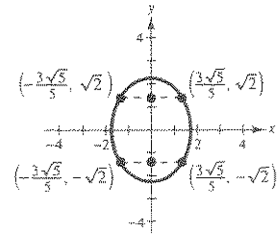
Length of latus recta:

$$\frac{2b^2}{a} = \frac{2 \cdot 2^2}{3} = \frac{8}{3}$$

Additional points: $\left(\pm\frac{4}{3}, -\sqrt{5}\right), \left(\pm\frac{4}{3}, \sqrt{5}\right)$

59. $5x^2 + 3y^2 = 15$

$$\frac{x^2}{3} + \frac{y^2}{5} = 1$$



$a = \sqrt{5}, b = \sqrt{3},$

$c = \sqrt{2}$

Points on the ellipse: $(\pm\sqrt{3}, 0), (0, \pm\sqrt{5})$

Length of latus recta: $\frac{2b^2}{a} = \frac{2 \cdot 3}{\sqrt{5}} = \frac{6\sqrt{5}}{5}$

Additional points: $\left(\pm\frac{3\sqrt{5}}{5}, -\sqrt{2}\right), \left(\pm\frac{3\sqrt{5}}{5}, \sqrt{2}\right)$

60. Answers will vary.

61. True. If $e \approx 1$ then the ellipse is elongated, not circular.

62. True. The ellipse is inside the circle.

63. (a) The length of the string is $2a$.

(b) The path is an ellipse because the sum of the distances from the two thumbtacks is always the length of the string, that is, it is constant.

64. (a) $a + b = 20 \implies b = 20 - a$

$A = \pi ab = \pi a(20 - a)$

(b) $264 = \pi a(20 - a)$

$\pi a^2 - 20\pi a + 264 = 0$

$a \approx 14$ or $a \approx 6$ by the Quadratic Formula

$b = 6 \qquad b = 14$

Since $a > b$, we choose $a = 14$ and $b = 6$.

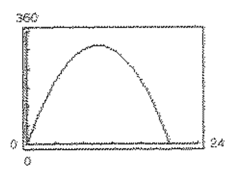
$$\frac{x^2}{14^2} + \frac{y^2}{6^2} = 1$$

$$\frac{x^2}{196} + \frac{y^2}{36} = 1$$

(c)

a	8	9	10	11	12	13
A	301.6	311.0	314.2	311.0	301.6	285.9

(d)



The area is maximum when $a = b = 10$ and it is a circle.

65. Center: $(6, 2)$

Foci: $(2, 2), (10, 2) \implies c = 4$

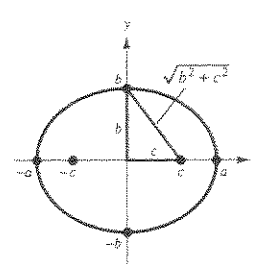
$(a + c) + (a - c) = 2a = 36 \implies a = 18$

$b^2 = a^2 - b^2 \implies b = \sqrt{18^2 - 16} = \sqrt{308}$

Horizontal major axis

$$\frac{(x - 6)^2}{324} + \frac{(y - 2)^2}{308} = 1$$

66. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$



The sum of the distances from any point on the ellipse to the two foci is constant. Using the vertex $(a, 0)$, you have

$(a + c) + (a - c) = 2a.$

From the figure,

$2\sqrt{b^2 + c^2} = 2a \implies a^2 = b^2 + c^2.$

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67. Arithmetic: $d = -11$

68. Geometric: $r = \frac{1}{2}$

69. Geometric: $r = 2$

70. Arithmetic: $d = 1$

71. $\sum_{n=0}^{6} 3^n = 1093$

72. $\sum_{n=0}^{6} (-3)^n = 547$

73. $\sum_{n=1}^{10} 4\left(\frac{3}{4}\right)^{n-1} \approx 15.099$

74. $\sum_{n=0}^{10} 5\left(\frac{4}{3}\right)^n \approx 340.155$

Section 9.3 Hyperbolas

- A **hyperbola** is the set of all points (x, y) the difference of whose distances from two distinct fixed points **(foci)** is constant.
- The standard equation of a hyperbola with center (h, k) and transverse and conjugate axes of lengths $2a$ and $2b$ is:
  (a) $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ if the transverse axis is horizontal.
  (b) $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$ if the transverse axis is vertical.
- $c^2 = a^2 + b^2$ where c is the distance from the center to a focus.
- The asymptotes of a hyperbola are:
  (a) $y = k + \frac{b}{a}(x - h)$ if the transverse axis is horizontal.
  (b) $y = k \pm \frac{a}{b}(x - h)$ the transverse axis is vertical.
- The eccentricity of a hyperbola is $e = \frac{c}{a}$.
- To classify a nondegenerate conic from its general equation $Ax^2 + Cy^2 + Dx + Ey + F = 0$:
  (a) If $A = C$ $(A \neq 0, C \neq 0)$, then it is a circle.
  (b) If $AC = 0$ $(A = 0$ or $C = 0$, but not both), then it is a parabola.
  (c) If $AC > 0$, then it is an ellipse.
  (d) If $AC < 0$, then it is a hyperbola.

Vocabulary Check

1. hyperbola
2. branches
3. transverse axis, center
4. asymptotes
5. $Ax^2 + Cy^2 + Dx + Ey + F = 0$

1. Center: $(0, 0)$

$a = 3, b = 5, c = \sqrt{34}$

Vertical transverse axis

Matches graph (b).

2. Center: $(0, 0)$

$a = 5, b = 3$

Vertical transverse axis

Matches graph (c).

3. Center: $(1, 0)$

$a = 4, b = 2$

Horizontal transverse axis

Matches graph (a).

4. Center: $(-1, 2)$

$a = 4, b = 3$

Horizontal transverse axis

Matches graph (d).

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5. $x^2 - y^2 = 1$

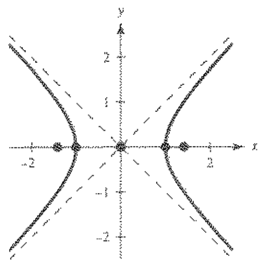
$a = 1, b = 1, c = \sqrt{2}$

Center: $(0, 0)$

Vertices: $(\pm 1, 0)$

Foci: $(\pm\sqrt{2}, 0)$

Asymptotes: $y = \pm x$



6. $\dfrac{x^2}{9} - \dfrac{y^2}{25} = 1$

Center: $(0, 0)$

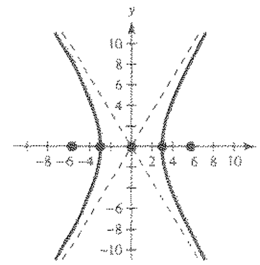
$a = 3, b = 5,$

$c = \sqrt{3^2 + 5^2} = \sqrt{34}$

Vertices: $(\pm 3, 0)$

Foci: $(\pm\sqrt{34}, 0)$

Asymptotes: $y = \pm\dfrac{b}{a}x = \pm\dfrac{5}{3}x$



7. $\dfrac{y^2}{1} - \dfrac{x^2}{4} = 1$

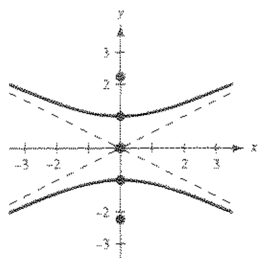
$a = 1, b = 2, c = \sqrt{5}$

Center: $(0, 0)$

Vertices: $(0, \pm 1)$

Foci: $(0, \pm\sqrt{5})$

Asymptotes: $y = \pm\dfrac{1}{2}x$



8. $\dfrac{y^2}{9} - \dfrac{x^2}{1} = 1$

$a = 3, b = 1,$

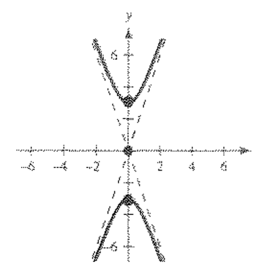
$c = \sqrt{3^2 + 1^2} = \sqrt{10}$

Center: $(0, 0)$

Vertices: $(0, \pm 3)$

Foci: $(0, \pm\sqrt{10})$

Asymptotes: $y = \pm 3x$



9. $\dfrac{y^2}{25} - \dfrac{x^2}{81} = 1$

$a = 5, b = 9, c = \sqrt{a^2 + b^2} = \sqrt{106}$

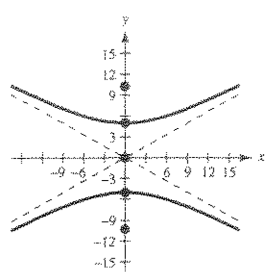
Center: $(0, 0)$

Vertices: $(0, \pm 5)$

Foci: $(0, \pm\sqrt{106})$

Asymptotes:

$y = \pm\dfrac{a}{b}x = \pm\dfrac{5}{9}x$



10. $\dfrac{x^2}{36} - \dfrac{y^2}{4} = 1$

$a = 6, b = 2,$

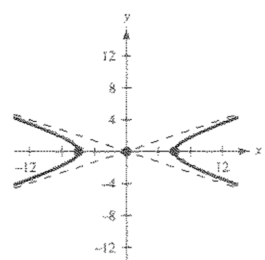
$c = \sqrt{36 + 4} = 2\sqrt{10}$

Center: $(0, 0)$

Vertices: $(\pm 6, 0)$

Foci: $(\pm 2\sqrt{10}, 0)$

Asymptotes: $y = \pm\dfrac{1}{3}x$



11. $\dfrac{(x-1)^2}{4} - \dfrac{(y+2)^2}{1} = 1$

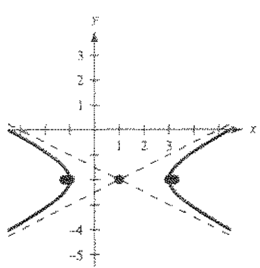
$a = 2, b = 1, c = \sqrt{5}$

Center: $(1, -2)$

Vertices:
$(-1, -2), (3, -2)$

Foci: $(1 \pm \sqrt{5}, -2)$

Asymptotes: $y = -2 \pm \dfrac{1}{2}(x - 1)$



12. $\dfrac{(x+3)^2}{144} - \dfrac{(y-2)^2}{25} = 1$

Center: $(-3, 2)$

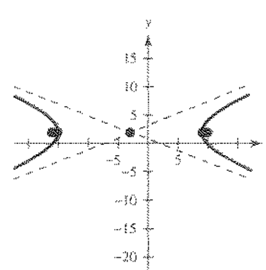
$a = 12, b = 5, c = 13$

Vertices: $(-15, 2), (9, 2)$

Foci: $(-16, 2), (10, 2)$

Asymptotes:

$y = 2 \pm \dfrac{5}{12}(x + 3)$



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13. $\dfrac{(y+5)^2}{1/9} - \dfrac{(x-1)^2}{1/4} = 1$

$a = \dfrac{1}{3}, b = \dfrac{1}{2}, c = \sqrt{\dfrac{1}{9} + \dfrac{1}{4}} = \dfrac{\sqrt{13}}{6}$

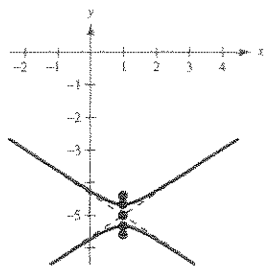
Center: $(1, -5)$

Vertices: $\left(1, -5 \pm \dfrac{1}{3}\right)$: $\left(1, -\dfrac{16}{3}\right), \left(1, -\dfrac{14}{3}\right)$

Foci: $\left(1, -5 \pm \dfrac{\sqrt{13}}{6}\right)$

Asymptotes: $y = k \pm \dfrac{a}{b}(x - h)$

$$y = -5 \pm \frac{2}{3}(x - 1)$$



14. $\dfrac{(y-1)^2}{1/4} - \dfrac{(x+3)^2}{1/16} = 1$

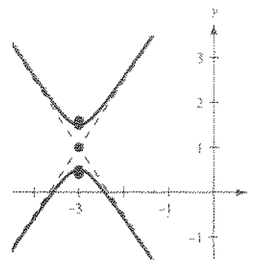
Center: $(-3, 1)$

$a = \dfrac{1}{2}, b = \dfrac{1}{4}, c = \sqrt{\dfrac{1}{4} + \dfrac{1}{16}} = \dfrac{\sqrt{5}}{4}$

Vertices: $\left(-3, \dfrac{1}{2}\right), \left(-3, \dfrac{3}{2}\right)$

Foci: $\left(-3, 1 \pm \dfrac{\sqrt{5}}{4}\right)$

Asymptotes: $y = 1 \pm \dfrac{1/2}{1/4}(x + 3) = 1 \pm 2(x + 3)$



15. (a) $4x^2 - 9y^2 = 36$

$\dfrac{x^2}{9} - \dfrac{y^2}{4} = 1$

(b) Center: $(0, 0)$

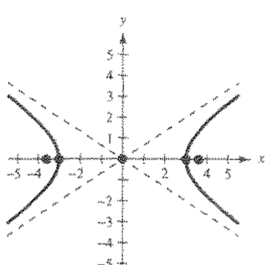
$a = 3, b = 2, c = \sqrt{9 + 4} = \sqrt{13}$

Vertices: $(\pm 3, 0)$

Foci: $(\pm\sqrt{13}, 0)$

Asymptotes: $y = \pm\dfrac{b}{a}x = \pm\dfrac{2}{3}x$

(c)



16. (a) $25x^2 - 4y^2 = 100$

$\dfrac{x^2}{4} - \dfrac{y^2}{25} = 1$

(b) Center: $(0, 0)$

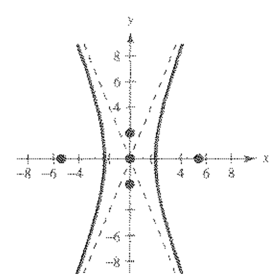
$a = 2, b = 5, c = \sqrt{4 + 25} = \sqrt{29}$

Vertices: $(\pm 2, 0)$

Foci: $(\pm\sqrt{29}, 0)$

Asymptotes: $y = \pm\dfrac{b}{a}x = \pm\dfrac{5}{2}x$

(c)



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17. (a) $2x^2 - 3y^2 = 6$

$$\frac{x^2}{3} - \frac{y^2}{2} = 1$$

(b) $a = \sqrt{3}, b = \sqrt{2}, c = \sqrt{5}$

Center: $(0, 0)$

Vertices: $(\pm\sqrt{3}, 0)$

Foci: $(\pm\sqrt{5}, 0)$

Asymptotes: $y = \pm\sqrt{\frac{2}{3}}x$

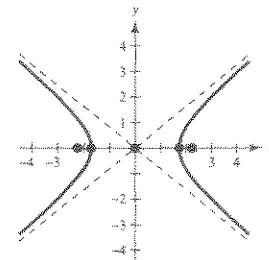
$$= \pm\frac{\sqrt{6}}{3}x$$

(c) To use a graphing calculator, solve first for y.

$$y^2 = \frac{2x^2 - 6}{3}$$

$$\left.\begin{aligned} y_1 &= \sqrt{\frac{2x^2 - 6}{3}} \\ y_2 &= -\sqrt{\frac{2x^2 - 6}{3}} \end{aligned}\right\} \text{Hyperbola}$$

$$\left.\begin{aligned} y_3 &= \sqrt{\frac{2}{3}}x \\ y_4 &= -\sqrt{\frac{2}{3}}x \end{aligned}\right\} \text{Asymptotes}$$



18. (a) $6y^2 - 3x^2 = 18$

$$\frac{y^2}{3} - \frac{x^2}{6} = 1$$

(b) $a = \sqrt{3}, b = \sqrt{6}, c = 3$

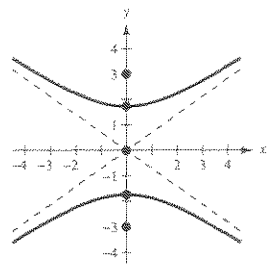
Center: $(0, 0)$

Vertices: $(0, \pm\sqrt{3})$

Foci: $(0, \pm 3)$

Asymptotes: $y = \pm\frac{\sqrt{3}}{\sqrt{6}}x = \pm\frac{\sqrt{2}}{2}x$

(c)



19. (a)

$$9x^2 - y^2 - 36x - 6y + 18 = 0$$

$$9(x^2 - 4x + 4) - (y^2 + 6y + 9) = -18 + 36 - 9$$

$$\frac{(x-2)^2}{1} - \frac{(y+3)^2}{9} = 1$$

(b) $a = 1, b = 3, c = \sqrt{10}$

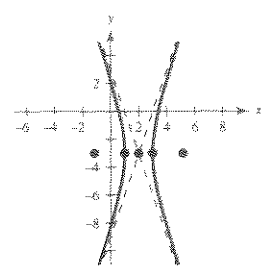
Center: $(2, -3)$

Vertices: $(1, -3), (3, -3)$

Foci: $(2 \pm \sqrt{10}, -3)$

Asymptotes: $y = -3 \pm 3(x - 2)$

(c)



20. (a) $x^2 - 9y^2 + 36y - 72 = 0$

$$x^2 - 9(y^2 - 4y + 4) = 72 - 36$$

$$x^2 - 9(y - 2)^2 = 36$$

$$\frac{x^2}{36} - \frac{(y-2)^2}{4} = 1$$

(b) $a = 6, b = 2,$

$c = \sqrt{36 + 4} = 2\sqrt{10}$

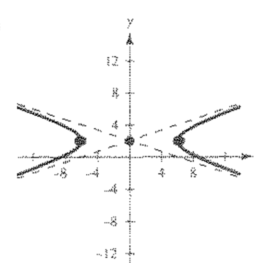
Center: $(0, 2)$

Vertices: $(\pm 6, 2)$

Foci: $(\pm 2\sqrt{10}, 2)$

Asymptotes: $y = 2 \pm \frac{1}{3}x$

(c)



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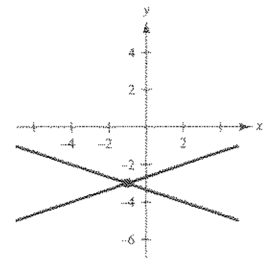
21. (a)

$$x^2 - 9y^2 + 2x - 54y - 80 = 0$$
$$(x^2 + 2x + 1) - 9(y^2 + 6y + 9) = 80 + 1 - 81$$
$$(x + 1)^2 - 9(y + 3)^2 = 0$$
$$y + 3 = \pm\tfrac{1}{3}(x + 1)$$

(b) Degenerate hyperbola is two lines intersecting at $(-1, -3)$.

(c)

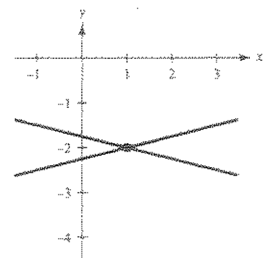


22. (a)

$$16y^2 - x^2 + 2x + 64y + 63 = 0$$
$$16(y^2 + 4y + 4) - (x^2 - 2x + 1) = -63 + 64 - 1$$
$$16(y + 2)^2 - (x - 1) = 0$$
$$y + 2 = \pm\tfrac{1}{4}(x - 1)$$

(b) Degenerate hyperbola is two intersecting lines at $(1, -2)$.

(c)



23. (a)

$$9y^2 - x^2 + 2x + 54y + 62 = 0$$
$$9(y^2 + 6y + 9) - (x^2 - 2x + 1) = -62 - 1 + 81$$
$$\frac{(y + 3)^2}{2} - \frac{(x - 1)^2}{18} = 1$$

(b) $a = \sqrt{2}, b = 3\sqrt{2}, c = 2\sqrt{5}$

Center: $(1, -3)$

Vertices: $\left(1, -3 \pm \sqrt{2}\right)$

Foci: $\left(1, -3 \pm 2\sqrt{5}\right)$

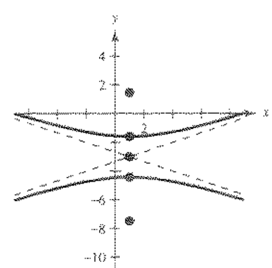
Asymptotes: $y = -3 \pm \dfrac{1}{3}(x - 1)$

(c) To use a graphing calculator, solve for y first.

$$9(y + 3)^2 = 18 + (x - 1)^2$$
$$y = -3 \pm \sqrt{\frac{18 + (x - 1)^2}{9}}$$

$$\left.\begin{aligned} y_1 &= -3 + \frac{1}{3}\sqrt{18 + (x - 1)^2} \\ y_2 &= -3 - \frac{1}{3}\sqrt{18 + (x - 1)^2} \end{aligned}\right\} \text{Hyperbola}$$

$$\left.\begin{aligned} y_3 &= -3 + \frac{1}{3}(x - 1) \\ y_4 &= -3 - \frac{1}{3}(x - 1) \end{aligned}\right\} \text{Asymptotes}$$



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24. (a)
$$9x^2 - y^2 + 54x + 10y + 55 = 0$$
$$9(x^2 + 6x + 9) - (y^2 - 10y + 25) = -55 + 81 - 25$$
$$\frac{(x+3)^2}{1/9} - \frac{(y-5)^2}{1} = 1$$

(b) $a = \frac{1}{3}, b = 1, c = \frac{\sqrt{10}}{3}$

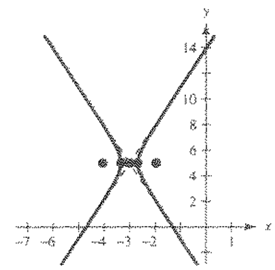
Center: $(-3, 5)$

Vertices: $\left(-3 \pm \frac{1}{3}, 5\right)$

Foci: $\left(-3 \pm \frac{\sqrt{10}}{3}, 5\right)$

Asymptotes: $y = 5 \pm 3(x + 3)$

(c)



25. Vertices: $(0, \pm 2) \implies a = 2$

Foci: $(0, \pm 4) \implies c = 4$

$b^2 = c^2 - a^2 = 16 - 4 = 12$

Center: $(0, 0) = (h, k)$

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$
$$\frac{y^2}{4} - \frac{x^2}{12} = 1$$

26. Vertices: $(\pm 3, 0) \implies a = 3$

Foci: $(\pm 6, 0) \implies c = 6$

$b^2 = c^2 - a^2 = 36 - 9 = 27$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$
$$\frac{x^2}{9} - \frac{y^2}{27} = 1$$

27. Vertices: $(\pm 1, 0) \implies a = 1$

Asymptotes:

$$y = \pm 5x \implies \frac{b}{a} = 5$$
$$\implies b = 5$$

Center: $(0, 0)$

$$\frac{x^2}{1} - \frac{y^2}{25} = 1$$

28. Vertices: $(0, \pm 3) \implies a = 3$

Asymptotes: $y = \pm 3x \implies \frac{a}{b} = 3, b = 1$

Center: $(0, 0) = (h, k)$

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$
$$\frac{y^2}{9} - x^2 = 1$$

29. Foci: $(0, \pm 8) \implies c = 8$

Asymptotes: $y = \pm 4x \implies \frac{a}{b} = 4 \implies a = 4b$

Center: $(0, 0) = (h, k)$

$c^2 = a^2 + b^2 \implies 64 = 16b^2 + b^2$

$$\frac{64}{17} = b^2 \implies a^2 = \frac{1024}{17}$$
$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$
$$\frac{y^2}{1024/17} - \frac{x^2}{64/17} = 1$$
$$\frac{17y^2}{1024} - \frac{17x^2}{64} = 1$$

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30. Foci: $(\pm 10, 0) \implies c = 10$

Asymptotes: $y = \pm\frac{3}{4}x \implies \frac{b}{a} = \frac{3m}{4m}$

$$c^2 = a^2 + b^2 \implies 100 = (3m)^2 + (4m)^2$$
$$100 = 25m^2$$
$$2 = m$$

$a = 4(2) = 8,\ b = 3(2) = 6$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{64} - \frac{y^2}{36} = 1$$

31. Vertices: $(2, 0), (6, 0) \implies a = 2$

Foci: $(0, 0), (8, 0) \implies c = 4$

$b^2 = c^2 - a^2 = 16 - 4 = 12$

Center: $(4, 0) = (h, k)$

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

$$\frac{(x-4)^2}{4} - \frac{y^2}{12} = 1$$

32. Vertices: $(2, 3), (2, -3) \implies a = 3$

Center: $(2, 0)$

Foci: $(2, 5), (2, -5) \implies c = 5$

$b^2 = c^2 - a^2 = 25 - 9 = 16$

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

$$\frac{y^2}{9} - \frac{(x-2)^2}{16} = 1$$

33. Vertices: $(4, 1), (4, 9) \implies a = 4$

Foci: $(4, 0), (4, 10) \implies c = 5$

$b^2 = c^2 - a^2 = 25 - 16 = 9$

Center: $(4, 5) = (h, k)$

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

$$\frac{(y-5)^2}{16} - \frac{(x-4)^2}{9} = 1$$

34. Vertices: $(-2, 1), (2, 1) \implies a = 2$

Center: $(0, 1)$

Foci: $(-3, 1), (3, 1) \implies c = 3$

$b^2 = c^2 - a^2 = 9 - 4 = 5$

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

$$\frac{x^2}{4} - \frac{(y-1)^2}{5} = 1$$

35. Vertices: $(2, 3), (2, -3) \implies a = 3$

Solution point: $(0, 5)$

Center: $(2, 0) = (h, k)$

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

$$\frac{y^2}{9} - \frac{(x-2)^2}{b^2} = 1 \implies$$

$$b^2 = \frac{9(x-2)^2}{y^2 - 9}$$
$$= \frac{9(-2)^2}{25 - 9} = \frac{36}{16} = \frac{9}{4}$$

$$\frac{y^2}{9} - \frac{(x-2)^2}{9/4} = 1$$

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36. Center: $(0, 1)$, $a = 2$

$$\frac{x^2}{4} - \frac{(y-1)^2}{b^2} = 1$$

Solution point: $(5, 4)$

$$\frac{25}{4} - \frac{9}{b^2} = 1$$

$$\frac{9}{b^2} = \frac{21}{4}$$

$$b^2 = \frac{36}{21} = \frac{12}{7}$$

$$\frac{x^2}{4} - \frac{(y-1)^2}{12/7} = 1$$

37. Vertices: $(0, 4), (0, 0)$

Center: $(0, 2)$, $a = 2$

$$\frac{(y-2)^2}{4} - \frac{x^2}{b^2} = 1$$

Passes through $(\sqrt{5}, -1)$

$$\frac{(-1-2)^2}{4} - \frac{5}{b^2} = 1$$

$$\frac{9}{4} - 1 = \frac{5}{b^2}$$

$$b^2 = 4 \implies b = 2$$

$$\frac{(y-2)^2}{4} - \frac{x^2}{4} = 1$$

38. Center: $(1, 0)$, $a = 2$

$$\frac{y^2}{4} - \frac{(x-1)^2}{b^2} = 1$$

Solution point: $(0, \sqrt{5})$

$$\frac{5}{4} - \frac{1}{b^2} = 1$$

$$\frac{1}{b^2} = \frac{1}{4} \implies b = 2$$

$$\frac{y^2}{4} - \frac{(x-1)^2}{4} = 1$$

39. Vertices:

$(1, 2), (3, 2) \implies a = 1$

Center: $(2, 2)$

Asymptotes:

$y = x, y = 4 - x$

$$\frac{b}{a} = 1 \implies b = 1$$

$$\frac{(x-2)^2}{1} - \frac{(y-2)^2}{1} = 1$$

40. Center: $(3, -3)$, $a = 3$

Asymptotes:

$y = x - 6, y = -x$

$$1 = \frac{a}{b} = \frac{3}{b} \implies b = 3$$

$$\frac{(y+3)^2}{9} - \frac{(x-3)^2}{9} = 1$$

41. Vertices: $(0, 2), (6, 2) \implies a = 3$

Asymptotes: $y = \frac{2}{3}x, y = 4 - \frac{2}{3}x$

$$\frac{b}{a} = \frac{2}{3} \implies b = 2$$

Center: $(3, 2) = (h, k)$

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

$$\frac{(x-3)^2}{9} - \frac{(y-2)^2}{4} = 1$$

42. Vertices: $(3, 0), (3, 4) \implies a = 2$

Asymptotes: $y = \frac{2}{3}x, y = 4 - \frac{2}{3}x$

$$\frac{a}{b} = \frac{2}{3} \implies b = 3$$

Center: $(3, 2) = (h, k)$

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

$$\frac{(y-2)^2}{4} - \frac{(x-3)^2}{9} = 1$$

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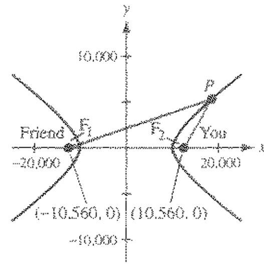
43. F_1: Friend's location $(-10{,}560, 0)$

F_2: Your location $(10{,}560, 0)$

$P(x, y)$: Location of lightning strike



$(1100)(18) = 19{,}800$

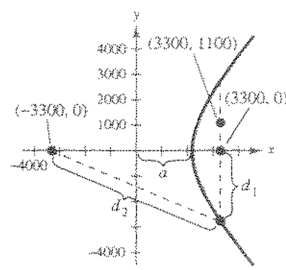
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$c = 10{,}560,\ a = \frac{19{,}800}{2} = 9900 \implies a^2 = 98{,}010{,}000$$

$$b^2 = c^2 - a^2 = 13{,}503{,}600$$

$$\frac{x^2}{98{,}010{,}000} - \frac{y^2}{13{,}503{,}600}$$

44. The explosion occurred on the vertical line through $(3300, 1100)$ and $(3300, 0)$.



$d_2 - d_1 = 4(1100) = 4400$

Hence,

$2a = 4400$

$a = 2200$

$c = 3300$

$b^2 = c^2 - a^2.$

The explosion occurred on the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1.$

Letting $x = 3300$,

$$y^2 = b^2\left(\frac{x^2}{a^2} - 1\right) = (3300^2 - 2200^2)\left(\frac{3300^2}{2200^2} - 1\right) \implies y = -2750.$$

$(3300, -2750)$

45. (a) $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$

$a = 1$; $(2, 9)$ is on the curve, so

$$\frac{4}{1} - \frac{81}{b^2} = 1 \implies \frac{81}{b^2} = 3$$

$$\implies b^2 = \frac{81}{3} \implies b = 3\sqrt{3}.$$

$$\frac{x^2}{1} - \frac{y^2}{27} = 1, \quad -9 \le y \le 9$$

(b) Because each unit is $\frac{1}{2}$ foot, 4 inches is $\frac{2}{3}$ of a unit. The base is 9 units from the origin, so

$$y = 9 - \frac{2}{3} = 8\frac{1}{3}.$$

When $y = \dfrac{25}{3}$,

$$x^2 = 1 + \frac{(25/3)^2}{27} \implies x \approx 1.88998.$$

So the width is $2x \approx 3.779956$ units, or 22.68 inches, or 1.88998 feet.

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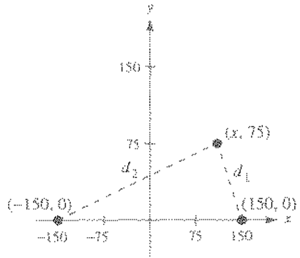
46. Foci: $(\pm 150, 0) \implies c = 150$

Center: $(0, 0)$



(a) $d_2 - d_1 = (186{,}000)(0.001)$

$= 186 \implies 2a = 186 \implies a = 93$

$b^2 = c^2 - a^2 = 150^2 - 93^2 = 13{,}851$

$$\frac{x^2}{93^2} - \frac{y^2}{13{,}851} = 1$$

$$x^2 = 93^2\left(1 + \frac{75^2}{13{,}851}\right) \approx 12{,}161.43$$

$x \approx 110.3$ miles

(b) $150 - 93 = 57$ miles

(c) Bay to Station 1: 30 miles

Bay to Station 2: 270 miles

$$\frac{(270 - 30)}{186{,}000} \approx 0.00129 \text{ second}$$

(d) In this case,

$d_2 - d_1 = 186{,}000(0.00129) \approx 239.94 \implies a \approx 120$

and $b^2 = c^2 - a^2 = 8100$. The hyperbola is

$$\frac{x^2}{120^2} - \frac{y^2}{90^2} = 1.$$

For $y = 60$, $x^2 = 20{,}800$ and $x \approx 144.2$.

Position: $(144.2, 60)$

47. Center: $(0, 0)$

Focus: $(24, 0)$

$b^2 = c^2 - a^2 = 24^2 - a^2 = 576 - a^2$

$$\frac{x^2}{a^2} - \frac{y^2}{576 - a^2} = 1$$

$$\frac{24^2}{a^2} - \frac{24^2}{576 - a^2} = 1$$

$$\frac{576}{a^2} - \frac{576}{576 - a^2} = 1$$

$576(576 - a^2) - 576a^2 = a^2(576 - a^2)$

$a^4 - 1728a^2 + 331{,}776 = 0$

$a \approx \pm 38.83$ or $a \approx \pm 14.83$

Since $a < c$ and $c = 24$, we choose $a = 14.83$. The vertex is approximate at $(14.83, 0)$.
[**Note:** By the Quadratic Formula, the exact value of a is $a = 12(\sqrt{5} - 1)$.]

48. $\dfrac{x^2}{25} - \dfrac{y^2}{16} = 1$

$a = 5, b = 4, c = \sqrt{25 + 16} = \sqrt{41}$

The camera is $5 + \sqrt{41}$ units from the mirror.

49. $9x^2 + 4y^2 - 18x + 16y - 119 = 0$

$A = 9, C = 4$

$AC = 36 > 0$, Ellipse

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50. $x^2 + y^2 - 4x - 6y - 23 = 0$

$A = C = 1$, Circle

51. $16x^2 - 9y^2 + 32x + 54y - 209 = 0$

$A = 16, C = -9$

$AC = 16(-9) < 0$, Hyperbola

52. $x^2 + 4x - 8y + 20 = 0$

$A = 1, C = 0$

$AC = 0$, Parabola

53. $y^2 + 12x + 4y + 28 = 0$

$C = 1, A = 0$

$AC = 0$, Parabola

54. $4x^2 + 25y^2 + 16x + 250y + 541 = 0$

$A = 4, C = 25$

$AC = 100 > 0$, Ellipse

55. $x^2 + y^2 + 2x - 6y = 0$

$A = C = 1$, Circle

56. $y^2 - x^2 + 2x - 6y - 8 = 0$

$A = -1, C = 1$

$AC < 0$, Hyperbola

57. $x^2 - 6x - 2y + 7 = 0$

$A = 1, C = 0, D = -6, \ E = -2, F = 7$

$AC = 0 \implies$ Parabola

58. $9x^2 + 4y^2 - 90x + 8y + 228 = 0$

$A = 9, C = 4$

$AC = 9(4) = 36 > 0 \implies$ Ellipse

59. True. $e = \dfrac{c}{a} = \dfrac{\sqrt{a^2 + b^2}}{a}$

60. False. $b \neq 0$ because it is in the denominator.

61. False. For example,

$x^2 - y^2 - 2x + 2y = 0$

$(x - 1)^2 - (y - 1)^2 = 0$

is the graph of two intersecting lines.

62. True. The asymptotes are

$y = \pm\dfrac{b}{a}x.$

If they intersect at right angles, then

$\dfrac{b}{a} = \dfrac{-1}{(-b/a)} = \dfrac{a}{b} \implies a = b.$

63. Let (x, y) be such that the difference of the distances from $(c, 0)$ and $(-c, 0)$ is $2a$ (again only deriving one of the forms).

$$\begin{aligned}
2a &= \left|\sqrt{(x + c)^2 + y^2} - \sqrt{(x - c) + y^2}\right| \\
2a + \sqrt{(x - c)^2 + y^2} &= \sqrt{(x + c)^2 + y^2} \\
4a^2 + 4a\sqrt{(x - c)^2 + y^2} + (x - c)^2 + y^2 &= (x + c)^2 + y^2 \\
4a\sqrt{(x - c)^2 + y^2} &= 4cx - 4a^2 \\
a\sqrt{(x - c)^2 + y^2} &= cx - a^2 \\
a^2(x^2 - 2cx + c^2 + y^2) &= c^2x^2 - 2a^2cx + a^4 \\
a^2(c^2 - a^2) &= (c^2 - a^2)x^2 - a^2y^2
\end{aligned}$$

Let $b^2 = c^2 - a^2$. Then $a^2b^2 = b^2x^2 - a^2y^2 \implies 1 = \dfrac{x^2}{a^2} - \dfrac{y^2}{b^2}.$

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64. Answers will vary. See Example 3.

65. $|d_2 - d_1| =$ constant by definition of hyperbola

At the point $(a, 0)$,

$|d_2 - d_1| = |(a + c) - (c - a)| = 2a.$

66. Center: $(6, 2)$

Horizontal transverse axis

Foci at $(2, 2)$ and $(10, 2) \implies c = 4.$

$(c + a) - (c - a) = 6 \implies a = 3$

$b^2 = c^2 - a^2 = 16 - 9 = 7$

$\frac{(x-6)^2}{9} - \frac{(y-2)^2}{7} = 1$

67. At the point $(a, 0)$, the difference of the distances to the foci $(\pm c, 0)$ is $(c + a) - (c - a) = 2a$.
Let (x, y) be a point on the hyperbola.

$$\begin{aligned}
2a &= \sqrt{(x+c)^2 + y^2} - \sqrt{(x-c)^2 + y^2} \\
2a + \sqrt{(x-c)^2 + y^2} &= \sqrt{(x+c)^2 + y^2} \\
4a^2 + 4a\sqrt{(x-c)^2 + y^2} + (x-c)^2 + y^2 &= (x+c)^2 + y^2 \\
4a\sqrt{(x-c)^2 + y^2} &= 4cx - 4a^2 \\
a\sqrt{(x-c)^2 + y^2} &= cx - a^2 \\
a^2(x^2 - 2cx + c^2 + y^2) &= c^2x^2 - 2a^2cx + a^4 \\
a^2(c^2 - a^2) &= (c^2 - a^2)x^2 - a^2y^2 \\
1 &= \frac{x^2}{a^2} - \frac{y^2}{c^2 - a^2}
\end{aligned}$$

Thus, $c^2 - a^2 = b^2$, as desired.

68. If $A = C \neq 0$, then by completing the square you obtain a circle.

If $A = 0$ and $C \neq 0$, then $Cy^2 + Dx + Ey + F = 0$ is a parabola (complete the square).
Same for $A \neq 0$ and $C = 0$.

If $AC > 0$, then both A and C are positive (or both negative). By completing the square you obtain an ellipse.

If $AC < 0$, then A and C have opposite signs. You obtain a hyperbola.

69. $(x^3 - 3x^2) - (6 - 2x - 4x^2) = x^3 + x^2 + 2x - 6$

70. $\left(3x - \frac{1}{2}\right)(x + 4) = 3x^2 + 12x - \frac{1}{2}x - 2$

$= 3x^2 + \frac{23}{2}x - 2$

71.
$$\begin{array}{r|rrrr}
-2 & 1 & 0 & -3 & 4 \\
 & & -2 & 4 & -2 \\
\hline
 & 1 & -2 & 1 & 2
\end{array}$$

$\frac{x^3 - 3x + 4}{x + 2} = x^2 - 2x + 1 + \frac{2}{x + 2}$

72. $[(x + y) + 3]^2 = (x + y)^2 + 6(x + y) + 9$

$= x^2 + 2xy + y^2 + 6x + 6y + 9$

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73. $x^3 - 16x = x(x^2 - 16) = x(x - 4)(x + 4)$

74. $x^2 + 14x + 49 = (x + 7)^2$

75. $2x^3 - 24x^2 + 72x = 2x(x^2 - 12x + 36)$
$= 2x(x - 6)^2$

76. $6x^3 - 11x^2 - 10x = x(6x^2 - 11x - 10)$
$= x(3x + 2)(2x - 5)$

77. $16x^3 + 54 = 2(8x^3 + 27)$
$= 2(2x + 3)(4x^2 - 6x + 9)$

78. $4 - x + 4x^2 - x^3 = (4 - x) + x^2(4 - x)$
$= (4 - x)(x^2 + 1)$
$= (4 - x)(x + i)(x - i)$

Section 9.4 Rotation and Systems of Quadratic Equations

- The general second-degree equation $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ can be rewritten $A'(x')^2 + C'(y')^2 + D'x' + E'y' + F' = 0$ by rotating the coordinate axes through the angle θ where $\cot 2\theta = (A - C)/B$.
- $x = x'\cos\theta - y'\sin\theta$
  $y = x'\sin\theta + y'\cos\theta$
- The graph of the nondegenerate equation $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ is:
  (a) An ellipse or circle if $B^2 - 4AC < 0$.
  (b) A parabola if $B^2 - 4AC = 0$.
  (c) A hyperbola if $B^2 - 4AC > 0$.

Vocabulary Check

1. rotation, axes **2.** invariant under rotation **3.** discriminant

1. $\theta = 90°$; Point: $(0, 3)$

$x = x'\cos\theta - y'\sin\theta$ $\quad$ $y = x'\sin\theta + y'\cos\theta$

$0 = x'\cos 90° - y'\sin 90°$ $\quad$ $3 = x'\sin 90° + y'\cos 90°$

$0 = y'$ $\quad$ $3 = x'$

Thus, $(x', y') = (3, 0)$.

2. $\theta = 45°$; Point: $(3, 3)$

$x = x'\cos\theta - y'\sin\theta$ $\quad$ $y = x'\sin\theta + y'\cos\theta$

$3 = x'\cos 45° - y'\sin 45°$ $\quad$ $3 = x'\sin 45° + y'\cos 45°$

$3 = \frac{\sqrt{2}}{2}x' - \frac{\sqrt{2}}{2}y'$ $\quad$ $3 = \frac{\sqrt{2}}{2}x' + \frac{\sqrt{2}}{2}y'$

Adding, $6 = \sqrt{2}x' \implies x' = \frac{6}{\sqrt{2}} = 3\sqrt{2}$.

Subtracting, $\sqrt{2}y' = 0 \implies y' = 0$.

Thus, $(x', y') = (3\sqrt{2}, 0)$.

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3. $xy + 1 = 0$

$A = 0, B = 1, C = 0$

$$\cot 2\theta = \frac{A - C}{B} = 0 \implies 2\theta = \frac{\pi}{2} \implies \theta = \frac{\pi}{4}$$

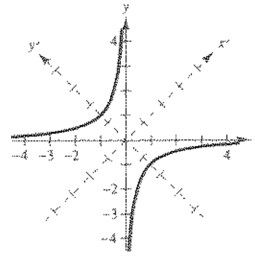
$$x = x'\cos\frac{\pi}{4} - y'\sin\frac{\pi}{4} = x'\left(\frac{\sqrt{2}}{2}\right) - y'\left(\frac{\sqrt{2}}{2}\right) = \frac{x' - y'}{\sqrt{2}}$$

$$y = x'\sin\frac{\pi}{4} + y'\cos\frac{\pi}{4} = x'\left(\frac{\sqrt{2}}{2}\right) + y'\left(\frac{\sqrt{2}}{2}\right) = \frac{x' + y'}{\sqrt{2}}$$

$$xy + 1 = 0$$

$$\left(\frac{x' - y'}{\sqrt{2}}\right)\left(\frac{x' + y'}{\sqrt{2}}\right) + 1 = 0$$

$$\frac{(y')^2}{2} - \frac{(x')^2}{2} = 1, \text{ Hyperbola}$$



4. $xy - 2 = 0, A = 0, B = 1, C = 0$

$$\cot 2\theta = \frac{A - C}{B} = 0 \implies 2\theta = \frac{\pi}{2} \implies \theta = \frac{\pi}{4}$$

$$x = x'\cos\frac{\pi}{4} - y'\sin\frac{\pi}{4} = x'\left(\frac{\sqrt{2}}{2}\right) - y'\left(\frac{\sqrt{2}}{2}\right) = \frac{x' - y'}{\sqrt{2}}$$

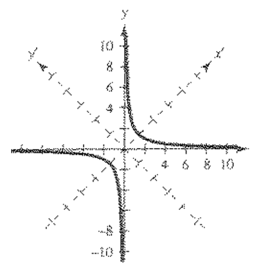
$$y = x'\sin\frac{\pi}{4} + y'\cos\frac{\pi}{4} = x'\left(\frac{\sqrt{2}}{2}\right) + y'\left(\frac{\sqrt{2}}{2}\right) = \frac{x' + y'}{\sqrt{2}}$$

$$xy - 2 = 0$$

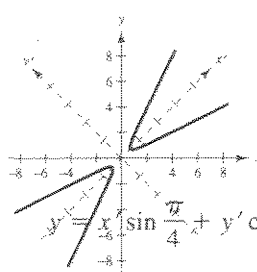
$$\left(\frac{x' - y'}{\sqrt{2}}\right)\left(\frac{x' + y'}{\sqrt{2}}\right) - 2 = 0$$

$$\frac{(x')^2 - (y')^2}{2} = 2$$

$$\frac{(x')^2}{4} - \frac{(y')^2}{4} = 1, \text{ Hyperbola}$$



5. $x^2 - 4xy + y^2 + 1 = 0$



$A = 1, B = -4, C = 1$

$$\cot 2\theta = \frac{A - C}{B} = 0 \implies 2\theta = \frac{\pi}{2} \implies \theta = \frac{\pi}{4}$$

$$x = x'\cos\frac{\pi}{4} - y'\sin\frac{\pi}{4}$$

$$= x'\left(\frac{\sqrt{2}}{2}\right) - y'\left(\frac{\sqrt{2}}{2}\right)$$

$$= \frac{\sqrt{2}}{2}(x' - y')$$

$$y = x'\sin\frac{\pi}{4} + y'\cos\frac{\pi}{4}$$

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5. —CONTINUED—

$$x^2 - 4xy + y^2 + 1 = 0$$

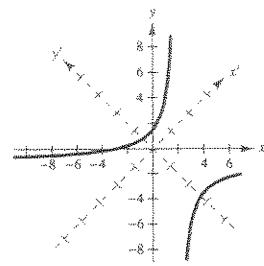
$$\left[\frac{\sqrt{2}}{2}(x' - y')\right]^2 - 4\left[\frac{\sqrt{2}}{2}(x' - y')\frac{\sqrt{2}}{2}(x' + y')\right] + \left[\frac{\sqrt{2}}{2}(x' + y')\right]^2 + 1 = 0$$

$$\frac{1}{2}(x')^2 - x'y' + \frac{1}{2}(y')^2 - 2[(x')^2 - (y')^2] + \frac{1}{2}(x')^2 + x'y' + \frac{1}{2}(y')^2 + 1 = 0$$

$$-(x')^2 + 3(y')^2 = -1$$

$$(x')^2 - \frac{(y')^2}{1/3} = 1, \text{ Hyperbola}$$

6. $xy + x - 2y + 3 = 0$



$A = 0, B = 1, C = 0$

$$\cot 2\theta = \frac{A - C}{B} = 0 \implies 2\theta = \frac{\pi}{2} \implies \theta = \frac{\pi}{4}$$

$$x = x'\cos\frac{\pi}{4} - y'\sin\frac{\pi}{4} \qquad y = x'\sin\frac{\pi}{4} + y'\cos\frac{\pi}{4}$$

$$= x'\left(\frac{\sqrt{2}}{2}\right) - y'\left(\frac{\sqrt{2}}{2}\right) \qquad = x'\left(\frac{\sqrt{2}}{2}\right) + y'\left(\frac{\sqrt{2}}{2}\right)$$

$$= \frac{x' - y'}{\sqrt{2}} \qquad = \frac{x' + y'}{\sqrt{2}}$$

$$xy + x - 2y + 3 = 0$$

$$\left(\frac{x' - y'}{\sqrt{2}}\right)\left(\frac{x' + y'}{\sqrt{2}}\right) + \left(\frac{x' - y'}{\sqrt{2}}\right) - 2\left(\frac{x' + y'}{\sqrt{2}}\right) + 3 = 0$$

$$\frac{(x')^2}{2} - \frac{(y')^2}{2} + \frac{x'}{\sqrt{2}} - \frac{y'}{\sqrt{2}} - \frac{2x'}{\sqrt{2}} - \frac{2y'}{\sqrt{2}} + 3 = 0$$

$$\left[(x')^2 - \sqrt{2}x' + \left(\frac{\sqrt{2}}{2}\right)^2\right] - \left[(y')^2 + 3\sqrt{2}y' + \left(\frac{3\sqrt{2}}{2}\right)^2\right] = -6 + \left(\frac{\sqrt{2}}{2}\right)^2 - \left(\frac{3\sqrt{2}}{2}\right)^2$$

$$\left(x' - \frac{\sqrt{2}}{2}\right)^2 - \left(y' + \frac{3\sqrt{2}}{2}\right)^2 = -10$$

$$\frac{\left(y' + \frac{3\sqrt{2}}{2}\right)^2}{10} - \frac{\left(x' - \frac{\sqrt{2}}{2}\right)^2}{10} = 1, \text{ Hyperbola}$$

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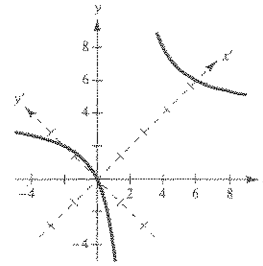
7. $xy - 2y - 4x = 0$

$A = 0, B = 1, C = 0$

$\cot 2\theta = \frac{A - C}{B} = 0 \implies 2\theta = \frac{\pi}{2} \implies \theta = \frac{\pi}{4}$



$$x = x'\cos\frac{\pi}{4} - y'\sin\frac{\pi}{4} \qquad y = x'\sin\frac{\pi}{4} + y'\cos\frac{\pi}{4}$$

$$= x'\left(\frac{\sqrt{2}}{2}\right) - y'\left(\frac{\sqrt{2}}{2}\right) \qquad = x'\left(\frac{\sqrt{2}}{2}\right) + y'\left(\frac{\sqrt{2}}{2}\right)$$

$$= \frac{x' - y'}{\sqrt{2}} \qquad = \frac{x' + y'}{\sqrt{2}}$$

$$xy - 2y - 4x = 0$$

$$\left(\frac{x' - y'}{\sqrt{2}}\right)\left(\frac{x' + y'}{\sqrt{2}}\right) - 2\left(\frac{x' + y'}{\sqrt{2}}\right) - 4\left(\frac{x' - y'}{\sqrt{2}}\right) = 0$$

$$\frac{(x')^2}{2} - \frac{(y')^2}{2} - \sqrt{2}x' - \sqrt{2}y' - 2\sqrt{2}x' + 2\sqrt{2}y' = 0$$

$$\left[(x')^2 - 6\sqrt{2}x' + \left(3\sqrt{2}\right)^2\right] - \left[(y')^2 - 2\sqrt{2}y' + \left(\sqrt{2}\right)^2\right] = 0 + \left(3\sqrt{2}\right)^2 - \left(\sqrt{2}\right)^2$$

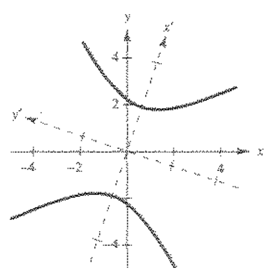
$$\left(x' - 3\sqrt{2}\right)^2 - \left(y' - \sqrt{2}\right)^2 = 16$$

$$\frac{\left(x' - 3\sqrt{2}\right)^2}{16} - \frac{\left(y' - \sqrt{2}\right)^2}{16} = 1, \quad \text{Hyperbola}$$

8. $2x^2 - 3xy - 2y^2 + 10 = 0$

$A = 2, B = -3, C = -2$

$\cot 2\theta = \frac{A - C}{B} = -\frac{4}{3} \implies \theta \approx 71.57°$



$\cos 2\theta = -\frac{4}{5}$

$$\sin\theta = \sqrt{\frac{1 - \cos 2\theta}{2}} = \sqrt{\frac{1 - (-4/5)}{2}} = \frac{3}{\sqrt{10}}$$

$$\cos\theta = \sqrt{\frac{1 + \cos 2\theta}{2}} = \sqrt{\frac{1 + (-4/5)}{2}} = \frac{1}{\sqrt{10}}$$

$$x = x'\cos\theta - y'\sin\theta \qquad y = x'\sin\theta + y'\cos\theta$$

$$= x'\left(\frac{1}{\sqrt{10}}\right) - y'\left(\frac{3}{\sqrt{10}}\right) \qquad = x'\left(\frac{3}{\sqrt{10}}\right) + y'\left(\frac{1}{\sqrt{10}}\right)$$

$$= \frac{x' - 3y'}{\sqrt{10}} \qquad = \frac{3x' + y'}{\sqrt{10}}$$

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8. —CONTINUED—

$$2x^2 - 3xy - 2y^2 + 10 = 0$$

$$2\left(\frac{x' - 3y'}{\sqrt{10}}\right)^2 - 3\left(\frac{x' - 3y'}{\sqrt{10}}\right)\left(\frac{3x' + y'}{\sqrt{10}}\right) - 2\left(\frac{3x' + y'}{\sqrt{10}}\right)^2 + 10 = 0$$

$$\frac{(x')^2}{5} - \frac{6x'y'}{5} + \frac{9(y')^2}{5} - \frac{9(x')^2}{10} + \frac{24x'y'}{10} + \frac{9(y')^2}{10} - \frac{9(x')^2}{5} - \frac{6x'y'}{5} - \frac{(y')^2}{5} + 10 = 0$$

$$-\frac{5}{2}(x')^2 + \frac{5}{2}(y')^2 = -10$$

$$\frac{(x')^2}{4} - \frac{(y')^2}{4} = 1, \text{ Hyperbola}$$

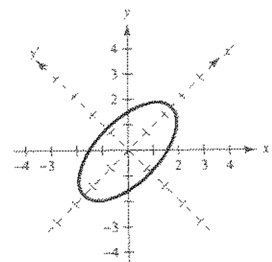
9. $5x^2 - 6xy + 5y^2 - 12 = 0$

$A = 5, B = -6, C = 5$

$\cot 2\theta = \dfrac{A - C}{B} = 0 \implies 2\theta = \dfrac{\pi}{2} \implies \theta = \dfrac{\pi}{4}$

$x = x' \cos \dfrac{\pi}{4} - y' \sin \dfrac{\pi}{4} = \dfrac{\sqrt{2}}{2}(x' - y')$

$y = x' \sin \dfrac{\pi}{4} + y' \cos \dfrac{\pi}{4} = \dfrac{\sqrt{2}}{2}(x' + y')$



$$5x^2 - 6xy + 5y^2 - 12 = 0$$

$$5\left[\frac{\sqrt{2}}{2}(x' - y')\right]^2 - 6\left[\frac{\sqrt{2}}{2}(x' - y')\frac{\sqrt{2}}{2}(x' + y')\right] + 5\left[\frac{\sqrt{2}}{2}(x' + y')\right]^2 = 12$$

$$\frac{5}{2}(x')^2 - 5x'y' + \frac{5}{2}(y')^2 - 3(x')^2 + 3(y')^2 + \frac{5}{2}(x')^2 + 5x'y' + \frac{5}{2}(y')^2 = 12$$

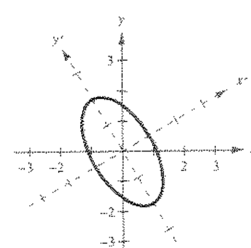
$$2(x')^2 + 8(y')^2 = 12$$

$$\frac{(x')^2}{6} + \frac{(y')^2}{3/2} = 1, \quad \text{Ellipse}$$

10. $13x^2 + 6\sqrt{3}xy + 7y^2 - 16 = 0$

$A = 13, B = 6\sqrt{3}, C = 7$

$\cot 2\theta = \dfrac{A - C}{B} = \dfrac{1}{\sqrt{3}} \implies 2\theta = \dfrac{\pi}{3} \implies \theta = \dfrac{\pi}{6}$



$x = x' \cos \dfrac{\pi}{6} - y' \sin \dfrac{\pi}{6}$ $\qquad$ $y = x' \sin \dfrac{\pi}{6} + y' \cos \dfrac{\pi}{6}$

$= x'\left(\dfrac{\sqrt{3}}{2}\right) - y'\left(\dfrac{1}{2}\right)$ $\qquad$ $= x'\left(\dfrac{1}{2}\right) + y'\left(\dfrac{\sqrt{3}}{2}\right)$

$= \dfrac{\sqrt{3}x' - y'}{2}$ $\qquad$ $= \dfrac{x' + \sqrt{3}y'}{2}$

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10. —CONTINUED—

$$13x^2 + 6\sqrt{3}xy + 7y^2 - 16 = 0$$

$$13\left(\frac{\sqrt{3}x' - y'}{2}\right)^2 + 6\sqrt{3}\left(\frac{\sqrt{3}x' - y'}{2}\right)\left(\frac{x' + \sqrt{3}y'}{2}\right) + 7\left(\frac{x' + \sqrt{3}y'}{2}\right)^2 - 16 = 0$$

$$\frac{39(x')^2}{4} - \frac{13\sqrt{3}x'y'}{2} + \frac{13(y')^2}{4} + \frac{18(x')^2}{4} + \frac{18\sqrt{3}x'y'}{4} - \frac{6\sqrt{3}x'y'}{4}$$

$$- \frac{18(y')^2}{4} + \frac{7(x')^2}{4} + \frac{7\sqrt{3}x'y'}{2} + \frac{21(y')^2}{4} - 16 = 0$$

$$16(x')^2 + 4(y')^2 = 16$$

$$\frac{(x')^2}{1} + \frac{(y')^2}{4} = 1, \text{ Ellipse}$$

11. $3x^2 - 2\sqrt{3}xy + y^2 + 2x + 2\sqrt{3}y = 0$

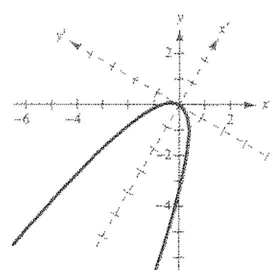
$A = 3, B = -2\sqrt{3}, C = 1$

$$\cot 2\theta = \frac{A - C}{B} = -\frac{1}{\sqrt{3}} \implies \theta = 60°$$

$$x = x' \cos 60° - y' \sin 60°$$

$$= x'\left(\frac{1}{2}\right) - y'\left(\frac{\sqrt{3}}{2}\right) = \frac{x' - \sqrt{3}y'}{2}$$

$$y = x' \sin \theta + y' \cos \theta = \frac{\sqrt{3}x' + y'}{2}$$



$$3x^2 - 2\sqrt{3}xy + y^2 + 2x + 2\sqrt{3}y = 0$$

$$3\left(\frac{x' - \sqrt{3}y'}{2}\right)^2 - 2\sqrt{3}\left(\frac{x' - \sqrt{3}y'}{2}\right)\left(\frac{\sqrt{3}x' + y'}{2}\right) + \left(\frac{\sqrt{3}x' + y'}{2}\right)^2 + 2\left(\frac{x' - \sqrt{3}y'}{2}\right)$$

$$+ 2\sqrt{3}\left(\frac{\sqrt{3}x' + y'}{2}\right) = 0$$

$$\frac{3(x')^2}{4} - \frac{6\sqrt{3}x'y'}{4} + \frac{9(y')^2}{4} - \frac{6(x')^2}{4} + \frac{4\sqrt{3}x'y'}{4} + \frac{6(y')^2}{4} + \frac{3(x')^2}{4} + \frac{2\sqrt{3}x'y'}{4} + \frac{(y')^2}{4}$$

$$+ x' - \sqrt{3}y' + 3x' + \sqrt{3}y' = 0$$

$$4(y')^2 + 4x' = 0$$

$$x' = -(y')^2, \quad \text{Parabola}$$

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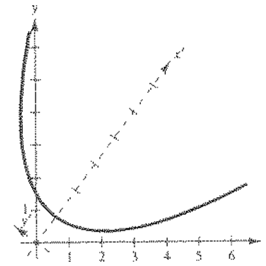
12. $16x^2 - 24xy + 9y^2 - 60x - 80y + 100 = 0$



$A = 16, B = -24, C = 9$

$$\cot 2\theta = \frac{A - C}{B} = -\frac{7}{24} \implies \theta \approx 53.13^\circ$$

$$\cos 2\theta = -\frac{7}{25}$$

$$\sin \theta = \sqrt{\frac{1 - \cos 2\theta}{2}} = \sqrt{\frac{1 - (-7/25)}{2}} = \frac{4}{5}$$

$$\cos \theta = \sqrt{\frac{1 + \cos 2\theta}{2}} = \sqrt{\frac{1 + (-7/25)}{2}} = \frac{3}{5}$$

$$x = x'\cos\theta - y'\sin\theta = x'\left(\frac{3}{5}\right) - y'\left(\frac{4}{5}\right) = \frac{3x' - 4y'}{5}$$

$$y = x'\sin\theta + y'\cos\theta = x'\left(\frac{4}{5}\right) + y'\left(\frac{3}{5}\right) = \frac{4x' + 3y'}{5}$$

$$16x^2 - 24xy + 9y^2 - 60x - 80y + 100 = 0$$

$$16\left(\frac{3x' - 4y'}{5}\right)^2 - 24\left(\frac{3x' - 4y'}{5}\right)\left(\frac{4x' + 3y'}{5}\right) + 9\left(\frac{4x' + 3y'}{5}\right)^2 - 60\left(\frac{3x' - 4y'}{5}\right) - 80\left(\frac{4x' + 3y'}{5}\right) + 100 = 0$$

$$\frac{144(x')^2}{25} - \frac{384x'y'}{25} + \frac{256(y')^2}{25} - \frac{288(x')^2}{25} + \frac{168x'y'}{25} + \frac{288(y')^2}{25} + \frac{144(x')^2}{25} + \frac{216x'y'}{25}$$

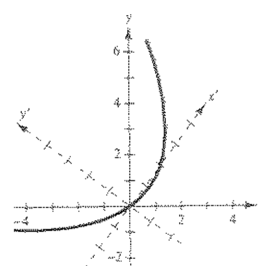
$$+ \frac{81(y')^2}{25} - 36x' + 48y' - 64x' - 48y' + 100 = 0$$

$$25(y')^2 - 100x' + 100 = 0$$

$$(y')^2 = 4(x' - 1)$$

Parabola

13. $9x^2 + 24xy + 16y^2 + 90x - 130y = 0$



$A = 9, B = 24, C = 16$

$$\cot 2\theta = \frac{A - C}{B} = -\frac{7}{24} \implies \theta \approx 53.13^\circ$$

$$\cos 2\theta = -\frac{7}{25}$$

$$\sin \theta = \sqrt{\frac{1 - \cos 2\theta}{2}} = \sqrt{\frac{1 - (-7/25)}{2}} = \frac{4}{5}$$

$$\cos \theta = \sqrt{\frac{1 + \cos 2\theta}{2}} = \sqrt{\frac{1 + (-7/25)}{2}} = \frac{3}{5}$$

$$x = x'\cos\theta - y'\sin\theta \qquad y = x'\sin\theta + y'\cos\theta$$

$$= x'\left(\frac{3}{5}\right) - y'\left(\frac{4}{5}\right) \qquad = x'\left(\frac{4}{5}\right) + y'\left(\frac{3}{5}\right)$$

$$= \frac{3x' - 4y'}{5} \qquad = \frac{4x' + 3y'}{5}$$

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13. —CONTINUED—

$$9x^2 + 24xy + 16y^2 + 90x - 130y = 0$$

$$9\left(\frac{3x' - 4y'}{5}\right)^2 + 24\left(\frac{3x' - 4y'}{5}\right)\left(\frac{4x' + 3y'}{5}\right) + 16\left(\frac{4x' + 3y'}{5}\right)^2 + 90\left(\frac{3x' - 4y'}{5}\right) - 130\left(\frac{4x' + 3y'}{5}\right) = 0$$

$$\frac{81(x')^2}{25} - \frac{216x'y'}{25} + \frac{144(y')^2}{25} + \frac{288(x')^2}{25} - \frac{168x'y'}{25} - \frac{288(y')^2}{25} + \frac{256(x')^2}{25} + \frac{384x'y'}{25} + \frac{144(y')^2}{25} + 54x' - 72y' - 104x' - 78y' = 0$$

$$25(x')^2 - 50x' - 150y' = 0$$

$$(x')^2 - 2x' + 1 = 6y' + 1$$

$$(x' - 1)^2 = 6\left(y' + \frac{1}{6}\right), \text{ Parabola}$$

14. $9x^2 + 24xy + 16y^2 + 80x - 60y = 0$

$A = 9, B = 24, C = 16$

$$\cot 2\theta = \frac{A - C}{B} = -\frac{7}{24} \implies \theta \approx 53.13^\circ$$

$$\cos 2\theta = -\frac{7}{25}$$

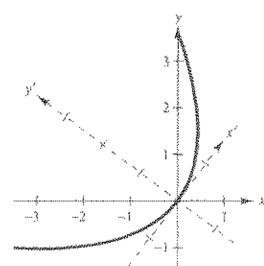
$$\sin\theta = \sqrt{\frac{1 - \cos 2\theta}{2}} = \sqrt{\frac{1 - (-7/25)}{2}} = \frac{4}{5}$$

$$\cos\theta = \sqrt{\frac{1 + \cos 2\theta}{2}} = \sqrt{\frac{1 + (-7/25)}{2}} = \frac{3}{5}$$

$x = x'\cos\theta - y'\sin\theta \qquad y = x'\sin\theta + y'\cos\theta$

$$x = x'\cos\theta - y'\sin\theta = x'\left(\frac{3}{5}\right) - y'\left(\frac{4}{5}\right) = \frac{3x' - 4y'}{5}$$

$$y = x'\sin\theta + y'\cos\theta = x'\left(\frac{4}{5}\right) + y'\left(\frac{3}{5}\right) = \frac{4x' + 3y'}{5}$$



$$9x^2 + 24xy + 16y^2 + 80x - 60y = 0$$

$$9\left(\frac{3x' - 4y'}{5}\right)^2 + 24\left(\frac{3x' - 4y'}{5}\right)\left(\frac{4x' + 3y'}{5}\right) + 16\left(\frac{4x' + 3y'}{5}\right)^2 + 80\left(\frac{3x' - 4y'}{5}\right) - 60\left(\frac{4x' + 3y'}{5}\right) = 0$$

$$\frac{81(x')^2}{25} - \frac{216x'y'}{25} + \frac{144(y')^2}{25} + \frac{288(x')^2}{25} - \frac{168x'y'}{25} - \frac{288(y')^2}{25} + \frac{256(x')^2}{25} + \frac{384x'y'}{25} + \frac{144(y')^2}{25} + 48x' - 64y' - 48x' - 36y' = 0$$

$$25(x')^2 - 100y' = 0$$

$$(x')^2 = 4y',$$

Parabola

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15. $x^2 + 3xy + y^2 = 20$

$$\cot 2\theta = \frac{A - C}{B} = \frac{1 - 1}{3} = 0 \implies \theta = \frac{\pi}{4} = 45^\circ$$

Solve for y in terms of x:

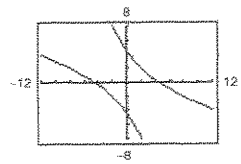
$$y^2 + 3xy = 20 - x^2$$

$$y^2 + 3xy + \frac{9x^2}{4} = 20 - x^2 + \frac{9x^2}{4}$$

$$\left(y + \frac{3}{2}x\right)^2 = 20 + \frac{5x^2}{4} = \frac{80 + 5x^2}{4}$$

$$y = -\frac{3}{2}x \pm \frac{\sqrt{80 + 5x^2}}{2}$$

Graph $y_1 = -\dfrac{3x}{2} + \dfrac{\sqrt{80 + 5x^2}}{2}$ and $y_2 = -\dfrac{3x}{2} - \dfrac{\sqrt{80 + 5x^2}}{2}$.



16. $x^2 - 4xy + 2y^2 = 8$

$A = 1, B = -4, C = 2$

$$\cot 2\theta = \frac{A - C}{B} = \frac{1 - 2}{-4} = \frac{1}{4}$$

$$\frac{1}{\tan 2\theta} = \frac{1}{4}$$

$$\tan 2\theta = 4$$

$$2\theta \approx 75.96^\circ$$

$$\theta \approx 37.98^\circ$$

To graph conic with a graphing calculator, we need to solve for y in terms of x.

$$x^2 - 4xy + 2y^2 = 8$$

$$y^2 - 2xy + x^2 = 4 - \frac{x^2}{2} + x^2$$

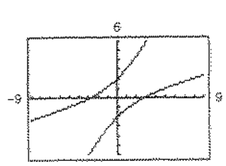
$$(y - x)^2 = 4 + \frac{x^2}{2}$$

$$y - x = \pm\sqrt{4 + \frac{x^2}{2}}$$

$$y = x \pm \sqrt{4 + \frac{x^2}{2}}$$

Graph $y_1 = x + \sqrt{4 + \dfrac{x^2}{2}}$ and

$y_2 = x - \sqrt{4 + \dfrac{x^2}{2}}$.



17. $17x^2 + 32xy - 7y^2 = 75$

$$\cot 2\theta = \frac{A - C}{B} = \frac{17 + 7}{32} = \frac{24}{32} = \frac{3}{4} \implies \theta \approx 26.57^\circ$$

Solve for y in terms of x by completing the square.

$$-7y^2 + 32xy = -17x^2 + 75$$

$$y^2 - \frac{32}{7}xy = \frac{17}{7}x^2 - \frac{75}{7}$$

$$y^2 - \frac{32}{7}xy + \frac{256}{49}x^2 = \frac{119}{49}x^2 - \frac{525}{49} + \frac{256}{49}x^2$$

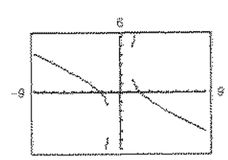
$$\left(y - \frac{16}{7}x\right)^2 = \frac{375x^2 - 525}{49}$$

$$y = \frac{16}{7}x \pm \sqrt{\frac{375x^2 - 525}{49}}$$

$$y = \frac{16x \pm 5\sqrt{15x^2 - 21}}{7}$$

Graph $y_1 = \dfrac{16x + 5\sqrt{15x^2 - 21}}{7}$ and

$y_2 = \dfrac{16x - 5\sqrt{15x^2 - 21}}{7}$.



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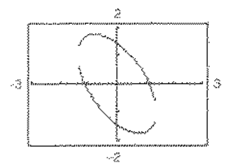
18. $40x^2 + 36xy + 25y^2 = 52$

$A = 40, B = 36, C = 25$

$$\cot 2\theta = \frac{A - C}{B} = \frac{40 - 25}{36} = \frac{5}{12}$$

$$\frac{1}{\tan 2\theta} = \frac{5}{12}$$



$$\tan 2\theta = \frac{12}{5}$$

$$2\theta \approx 67.38°$$

$$\theta \approx 33.69°$$

Solve for y in terms of x by completing the square:

$$25y^2 + 36xy = 52 - 40x^2$$

$$y^2 + \frac{36}{25}xy = \frac{52}{25} - \frac{40}{25}x^2$$

$$y^2 + \frac{36}{25}xy + \frac{324}{625}x^2 = \frac{52}{25} - \frac{40}{25}x^2 + \frac{324}{625}x^2$$

$$\left(y + \frac{18}{25}x\right)^2 = \frac{1300 - 676x^2}{625}$$

$$y + \frac{18}{25}x = \pm\sqrt{\frac{1300 - 676x^2}{625}}$$

$$y = \frac{-18x \pm \sqrt{1300 - 676x^2}}{25}$$

Graph $y_1 = \dfrac{-18x + \sqrt{1300 - 676x^2}}{25}$ and

$y_2 = \dfrac{-18x - \sqrt{1300 - 676x^2}}{25}$.

19. $32x^2 + 48xy + 8y^2 = 50$

$$\cot 2\theta = \frac{A - C}{B} = \frac{32 - 8}{48} = \frac{1}{2} \implies \theta \approx 31.72°$$

Solve for y in terms of x:

$$8y^2 + 48xy = -32x^2 + 50$$

$$y^2 + 6xy = -4x^2 + \frac{25}{4}$$

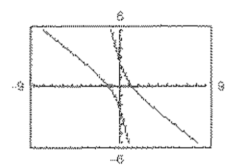
$$y^2 + 6xy + 9x^2 = -4x^2 + \frac{25}{4} + 9x^2$$

$$(y + 3x)^2 = 5x^2 + \frac{25}{4} = \frac{20x^2 + 25}{4}$$

$$y = -3x \pm \frac{\sqrt{20x^2 + 25}}{2}$$

Graph $y_1 = -3x + \dfrac{\sqrt{20x^2 + 25}}{2}$ and

$y_2 = -3x - \dfrac{\sqrt{20x^2 + 25}}{2}$.



20. $4x^2 - 12xy + 9y^2 + \left(4\sqrt{13} - 12\right)x - \left(6\sqrt{13} + 8\right)y = 91$

$A = 4, B = -12, C = 9$

$$\cot 2\theta = \frac{A - C}{B} = \frac{4 - 9}{-12} = \frac{5}{12}$$

$$\frac{1}{\tan 2\theta} = \frac{5}{12}$$

$$\tan 2\theta = \frac{12}{5}$$

$$2\theta \approx 67.38°$$

$$\theta \approx 33.69°$$

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20. —CONTINUED—

Solve for y in terms of x with the Quadratic Formula:

$$4x^2 - 12xy + 9y^2 + (4\sqrt{13} - 12)x - (6\sqrt{13} + 8)y = 91$$

$$9y^2 - (12x + 6\sqrt{13} + 8)y + (4x^2 + 4\sqrt{13}x - 12x - 91) = 0$$

$$a = 9, b = -(12x + 6\sqrt{13} + 8), c = 4x^2 + 4\sqrt{13}x - 12x - 91$$

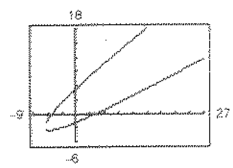
$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = \frac{(12x + 6\sqrt{13} + 8) \pm \sqrt{(12x + 6\sqrt{13} + 8)^2 - 4(9)(4x^2 + 4\sqrt{13}x - 12x - 91)}}{18}$$

$$= \frac{(12x + 6\sqrt{13} + 8) \pm \sqrt{624x + 3808 + 96\sqrt{13}}}{18}$$

Graph $y_1 = \dfrac{12x + 6\sqrt{13} + 8 + \sqrt{624x + 3808 + 96\sqrt{13}}}{18}$ and

$y_2 = \dfrac{12x + 6\sqrt{13} + 8 - \sqrt{624x + 3808 + 96\sqrt{13}}}{18}$.



21. $xy + 4 = 0$

$B^2 - 4AC = 1 \implies$ The graph is a hyperbola.

$\cot 2\theta = \dfrac{A - C}{B} = 0 \implies \theta = 45°$

Matches graph (e).

22. $x^2 + 2xy + y^2 = 0$

$(x + y)^2 = 0$

$x + y = 0$

$y = -x$

The graph is a line. Matches graph (b).

23. $-2x^2 + 3xy + 2y^2 + 3 = 0$

$B^2 - 4AC = (3)^2 - 4(-2)(2)$

$= 25 \implies$ The graph is a hyperbola.

$\cot 2\theta = \dfrac{A - C}{B} = -\dfrac{4}{3} \implies \theta \approx -18.43°$

Matches graph (f).

24. $x^2 - xy + 3y^2 - 5 = 0$

$A = 1, B = -1, C = 3$

$B^2 - 4AC = (-1)^2 - 4(1)(3) = -11$

The graph is an ellipse or circle.

$\cot 2\theta = \dfrac{A - C}{B} = \dfrac{1 - 3}{-1} = 2 \implies \theta \approx 13.28°$

Matches graph (a).

25. $3x^2 + 2xy + y^2 - 10 = 0$

$B^2 - 4AC = (2)^2 - 4(3)(1)$

$= -8 \implies$ The graph is an ellipse or circle.

$\cot 2\theta = \dfrac{A - C}{B} = 1 \implies \theta = 22.5°$

Matches graph (d).

26. $x^2 - 4xy + 4y^2 + 10x - 30 = 0$

$A = 1, B = -4, C = 4$

$B^2 - 4AC = (-4)^2 - 4(1)(4) = 0$

The graph is a parabola.

$\cot 2\theta = \dfrac{A - C}{B} = \dfrac{1 - 4}{-4} = \dfrac{3}{4} \implies \theta \approx 26.57°$

Matches graph (c).

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27. $16x^2 - 24xy + 9y^2 - 30x - 40y = 0$

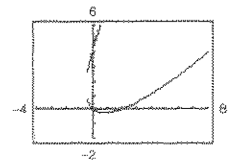
(a) $B^2 - 4AC = (-24)^2 - 4(16)(9) = 0 \implies$ Parabola

(b) $9y^2 - (24x + 40)y + (16x^2 - 30x) = 0$

$$y = \frac{(24x + 40) \pm \sqrt{(24x + 40)^2 - 4(9)(16x^2 - 30x)}}{2(9)}$$

$$= \frac{24x + 40 \pm \sqrt{3000x + 1600}}{18}$$

(c)



28. (a) $B^2 - 4AC = (-4)^2 - 4(1)(-2)$

$= 16 + 8 = 24$

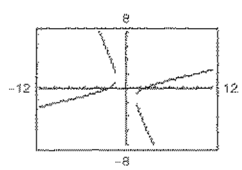
$> 0 \implies$ Hyperbola

(b) $-2y^2 - 4xy + x^2 - 6 = 0$

$$y = \frac{4x \pm \sqrt{16x^2 - 4(-2)(x^2 - 6)}}{-4}$$

$$= \frac{4x \pm \sqrt{24x^2 - 48}}{-4}$$

(c)



29. $15x^2 - 8xy + 7y^2 - 45 = 0$

(a) $B^2 - 4AC = (-8)^2 - 4(15)(7)$

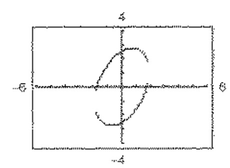
$= -356 \implies$ Ellipse or circle

(b) $7y^2 - 8xy + (15x^2 - 45) = 0$

$$y = \frac{8x \pm \sqrt{(-8x)^2 - 4(7)(15x^2 - 45)}}{14}$$

$$= \frac{8x \pm \sqrt{1260 - 356x^2}}{14}$$

(c)



30. (a) $B^2 - 4AC = 4^2 - 4(2)(5)$

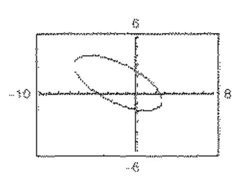
$= -24 < 0 \implies$ Ellipse or circle

(b) $5y^2 + (4x - 4)y + (2x^2 + 3x - 20) = 0$

$$y = \frac{(4 - 4x) \pm \sqrt{(4x - 4)^2 - 4(5)(2x^2 + 3x - 20)}}{10}$$

$$= \frac{(4 - 4x) \pm \sqrt{-24x^2 - 92x + 416}}{10}$$

(c)



31. $x^2 - 6xy - 5y^2 + 4x - 22 = 0$

(a) $B^2 - 4AC = (-6)^2 - 4(1)(-5)$

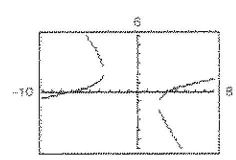
$= 56 \implies$ Hyperbola

(b) $-5y^2 - 6xy + (x^2 + 4x - 22) = 0$

$$y = \frac{6x \pm \sqrt{(-6x)^2 - 4(-5)(x^2 + 4x - 22)}}{-10}$$

$$= \frac{6x \pm \sqrt{56x^2 + 80x - 440}}{-10}$$

(c)



32. (a) $B^2 - 4AC = (-60)^2 - 4(36)(25)$

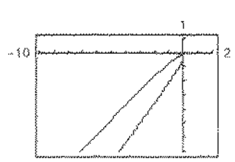
$= 0 \implies$ Parabola

(b) $25y^2 + (9 - 60x)y + 36x^2 = 0$

$$y = \frac{(60x - 9) \pm \sqrt{(9 - 60x)^2 - 100(36x^2)}}{50}$$

$$= \frac{60x - 9 \pm \sqrt{-1080x + 81}}{50}$$

(c)



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33. $x^2 + 4xy + 4y^2 - 5x - y - 3 = 0$

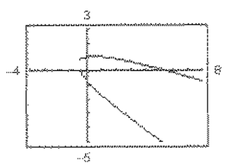
(a) $B^2 - 4AC = 4^2 - 4(1)(4) = 0 \implies$ Parabola

(b) $4y^2 + (4x - 1)y + (x^2 - 5x - 3) = 0$

$$y = \frac{(1 - 4x) \pm \sqrt{(4x - 1)^2 - 4(4)(x^2 - 5x - 3)}}{8}$$

$$= \frac{1 - 4x \pm \sqrt{72x + 49}}{8}$$

(c)



34. (a) $B^2 - 4AC = 1 - 4(1)(4)$

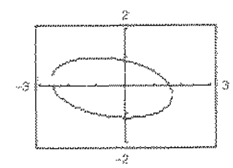
$= -15 < 0 \implies$ Ellipse or circle

(b) $4y^2 + (x + 1)y + (x^2 + x - 4) = 0$

$$y = \frac{-(x + 1) \pm \sqrt{(x + 1)^2 - 4(4)(x^2 + x - 4)}}{8}$$

$$= \frac{-x - 1 \pm \sqrt{-15x^2 - 14x + 65}}{8}$$

(c)

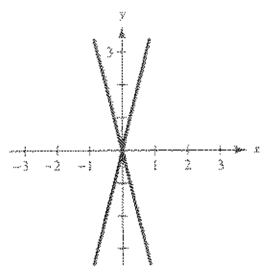


35. $y^2 - 16x^2 = 0$

$y^2 = 16x^2$

$y = \pm 4x$

Two intersecting lines

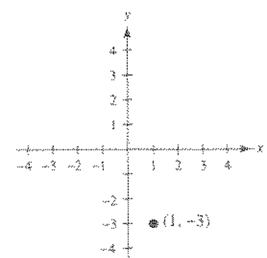


36. $x^2 + y^2 - 2x + 6y + 10 = 0$

$(x^2 - 2x + 1) + (y^2 + 6y + 9) = -10 + 1 + 9$

$(x - 1)^2 + (y + 3)^2 = 0$

Point at $(1, -3)$



37. $x^2 + 2xy + y^2 - 1 = 0$

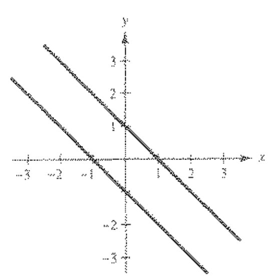
$(x + y)^2 - 1 = 0$

$(x + y)^2 = 1$

$x + y = \pm 1$

$y = -x \pm 1$

Two parallel lines



38. $x^2 - 10xy + y^2 = 0$

$y^2 - 10xy + 25x^2 = 25x^2 - x^2$

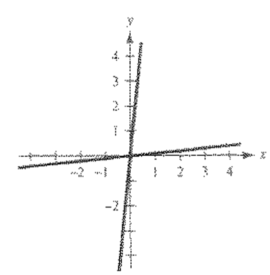
$(y - 5x)^2 = 24x^2$

$y - 5x = \pm\sqrt{24x^2}$

$y = 5x \pm 2\sqrt{6}x$

$y = (5 \pm 2\sqrt{6})x$

Two lines



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39. $x^2 + y^2 = 4$

$3x - y^2 = 0$

Adding:

$x^2 + 3x - 4 = 0$

$(x + 4)(x - 1) = 0 \implies x = 1, -4$

For $x = 1$, $y = \pm\sqrt{3}$.

$x = -4$ is impossible.

Solutions: $\left(1, \sqrt{3}\right), \left(1, -\sqrt{3}\right)$

40.
$$4x^2 + 9y^2 - 36y = 0$$
$$x^2 + y^2 - 27 = 0 \implies x^2 = 27 - y^2$$
$$4(27 - y^2) + 9y^2 - 36y = 0$$
$$5y^2 - 36y + 108 = 0$$
$$y = \frac{36 \pm \sqrt{36^2 - 4(5)(108)}}{10}$$
$$= \frac{36 \pm \sqrt{-864}}{10}$$

No solution

41.
$$\begin{aligned} -4x^2 - y^2 - 16x + 24y - 16 &= 0 \\ 4x^2 + y^2 + 40x - 24y + 208 &= 0 \\ \hline 24x + 192 &= 0 \\ 24x &= -192 \\ x &= -8 \end{aligned}$$

For $x = -8$:
$$\begin{aligned} -4(64) - y^2 - 16(-8) + 24y - 16 &= 0 \\ -y^2 + 24y - 144 &= 0 \\ y^2 - 24y + 144 &= 0 \\ (y - 12)^2 &= 0 \\ \implies y &= 12 \end{aligned}$$

Solution: $(-8, 12)$

42.
$$\begin{aligned} x^2 - 4y^2 - 20x - 64y - 172 &= 0 \implies (x - 10)^2 - 4(y + 8)^2 = 16 \\ 16x^2 + 4y^2 - 320x + 64y + 1600 &= 0 \implies 16(x - 10)^2 + 4(y + 8)^2 = 256 \\ \hline 17x^2 - 340x + 1428 &= 0 \\ (17x - 238)(x - 6) &= 0 \\ x = 6 \text{ or } x &= 14 \end{aligned}$$

When $x = 6$:
$$\begin{aligned} 6^2 - 4y^2 - 20(6) - 64y - 172 &= 0 \\ -4y^2 - 64y - 256 &= 0 \\ y^2 + 16y + 64 &= 0 \\ (y + 8)^2 &= 0 \\ y &= -8 \end{aligned}$$

When $x = 14$:
$$\begin{aligned} 14^2 - 4y^2 - 20(14) - 64y - 172 &= 0 \\ 4y^2 + 64y + 256 &= 0 \\ y^2 + 16y + 64 &= 0 \\ (y + 8)^2 &= 0 \\ y &= -8 \end{aligned}$$

Points of intersection: $(6, -8), (14, -8)$

43.
$$\begin{aligned} x^2 - y^2 - 12x + 16y - 64 &= 0 \\ x^2 + y^2 - 12x - 16y + 64 &= 0 \\ \hline 2x^2 - 24x &= 0 \\ x^2 - 12x &= 0 \\ x(x - 12) &= 0 \implies x = 0, 12 \end{aligned}$$

For $x = 0$:
$$\begin{aligned} -y^2 + 16y - 64 &= 0 \\ y^2 - 16y + 64 &= 0 \\ (y - 8)^2 &= 0 \implies y = 8 \end{aligned}$$

For $x = 12$:
$$\begin{aligned} 144 - y^2 - 12(12) + 16y - 64 &= 0 \\ -y^2 + 16y - 64 &= 0 \implies y = 8 \end{aligned}$$

Solutions: $(0, 8), (12, 8)$

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44. $$\begin{array}{r} x^2 + 4y^2 - 2x - 8y + 1 = 0 \\ -x^2 \qquad\quad + 2x - 4y - 1 = 0 \\ \hline 4y^2 \qquad - 12y \qquad = 0 \end{array} \quad \begin{array}{l} \Longrightarrow (x-1)^2 + 4(y-1)^2 = 4 \\ \Longrightarrow y = -\frac{1}{4}(x-1)^2 \\ \end{array}$$

$$4y(y-3) = 0$$

$$y = 0 \text{ or } y = 3$$

When $y = 0$:

$$x^2 + 4(0)^2 - 2x - 8(0) + 1 = 0$$
$$x^2 - 2x + 1 = 0$$
$$(x-1)^2 = 0$$
$$x = 1$$

When $y = 3$:

$$-x^2 + 2x - 4(3) - 1 = 0$$
$$x^2 - 2x + 13 = 0$$

No real solution

Point of intersection: $(1, 0)$

45. $$\begin{array}{r} -16x^2 - \quad y^2 + 24y - \ 80 = 0 \\ 16x^2 + 25y^2 \qquad\quad - 400 = 0 \\ \hline 24y^2 + \ 24y - 480 = 0 \end{array}$$

$$24(y+5)(y-4) = 0$$

$$y = -5 \text{ or } y = 4$$

When $y = -5$:

$$16x^2 + 25(-5)^2 - 400 = 0$$
$$16x^2 = -225$$

No real solution

When $y = 4$:

$$16x^2 + 25(4)^2 - 400 = 0$$
$$16x^2 = 0$$
$$x = 0$$

The point of intersection is $(0, 4)$. In standard form the equations are:

$$\frac{x^2}{4} + \frac{(y-12)^2}{64} = 1$$

$$\frac{x^2}{25} + \frac{y^2}{16} = 1$$

46. $$\begin{array}{r} 16x^2 - y^2 \qquad\quad + 16y - 128 = 0 \\ y^2 - 48x - 16y - \ 32 = 0 \\ \hline 16x^2 \qquad - 48x \qquad\quad - 160 = 0 \end{array} \quad \begin{array}{l} \Longrightarrow 16x^2 - (y-8)^2 = 64 \\ \Longrightarrow (y-8)^2 - 48x = 96 \\ \end{array}$$

$$16(x^2 - 3x - 10) = 0$$

$$(x-5)(x+2) = 0$$

$$x = 5 \text{ or } x = -2$$

When $x = 5$:

$$y^2 - 48(5) - 16y - 32 = 0$$
$$y^2 - 16y - 272 = 0$$
$$y = 8 \pm 4\sqrt{21}$$

When $x = -2$:

$$y^2 - 48(-2) - 16y - 32 = 0$$
$$y^2 - 16y + 64 = 0$$
$$(y-8)^2 = 0$$
$$y = 8$$

Points of intersection: $(5, 8 + 4\sqrt{21}), (5, 8 - 4\sqrt{21}), (-2, 8)$

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47. $2x^2 - y^2 + 6 = 0$

$$2x + y = 0 \implies y = -2x$$

$$2x^2 - (-2x)^2 + 6 = 0$$

$$-2x^2 + 6 = 0$$

$$x^2 = 3 \implies x = \pm\sqrt{3}$$

Two solutions: $(\sqrt{3}, -2\sqrt{3}), (-\sqrt{3}, 2\sqrt{3})$

48. $6x^2 + 3y^2 - 12 = 0$

$$x + y - 2 = 0 \implies y = 2 - x$$

$$6x^2 + 3(2 - x)^2 - 12 = 0$$

$$6x^2 + 3(4 - 4x + x^2) - 12 = 0$$

$$9x^2 - 12x = 0$$

$$3x(3x - 4) = 0$$

$$x = 0 \implies y = 2$$

$$x = \tfrac{4}{3} \implies y = 2 - \tfrac{4}{3} = \tfrac{2}{3}$$

Solutions: $(0, 2), \left(\frac{4}{3}, \frac{2}{3}\right)$

49. $10x^2 - 25y^2 - 100x + 160 = 0$

$$y^2 - 2x + 16 = 0 \implies y^2 = 2x - 16$$

$$10x^2 - 25(2x - 16) - 100x + 160 = 0$$

$$10x^2 - 150x + 560 = 0$$

$$x^2 - 15x + 56 = 0$$

$$(x - 8)(x - 7) = 0$$

$x = 8 \implies y^2 = 0 \implies (8, 0)$

$x = 7 \implies y^2 = -2$ impossible

One solution: $(8, 0)$

50. $4x^2 - y^2 - 8x + 6y - 9 = 0$

$2x^2 - 3y^2 + 4x + 18y - 43 = 0$

From Equation 1:

$y^2 - 6y = 4x^2 - 8x - 9$

$3y^2 - 18y = 12x^2 - 24x - 27$

In Equation 2:

$$2x^2 + 4x - 43 - (3y^2 - 18y) = 0$$

$$2x^2 + 4x - 43 - (12x^2 - 24x - 27) = 0$$

$$-10x^2 + 28x - 16 = 0$$

$$5x^2 - 14x + 8 = 0$$

$$(x - 2)(5x - 4) = 0$$

$x = 2 \implies y^2 - 6y = -9$

$\implies (y - 3)^2 = 0 \implies y = 3$

$x = \frac{4}{5} \implies y^2 - 6y = -\frac{321}{25}$ No solution

Solution: $(2, 3)$

51. $xy + x - 2y + 3 = 0 \implies y = \dfrac{-x - 3}{x - 2}$

$$x^2 + 4y^2 - 9 = 0$$

$$x^2 + 4\left(\frac{-x - 3}{x - 2}\right)^2 = 9$$

$$x^2(x - 2)^2 + 4(-x - 3)^2 = 9(x - 2)^2$$

$$x^2(x^2 - 4x + 4) + 4(x^2 + 6x + 9) = 9(x^2 - 4x + 4)$$

$$x^4 - 4x^3 + 4x^2 + 4x^2 + 24x + 36 = 9x^2 - 36x + 36$$

$$x^4 - 4x^3 - x^2 + 60x = 0$$

$$x(x + 3)(x^2 - 7x + 20) = 0$$

$$x = 0 \text{ or } x = -3$$

Note: $x^2 - 7x + 20 = 0$ has no real solution.

When $x = 0$: $y = \dfrac{-0 - 3}{0 - 2} = \dfrac{3}{2}$

When $x = -3$: $y = \dfrac{-(-3) - 3}{-3 - 2} = 0$

The points of intersection are $\left(0, \frac{3}{2}\right), (-3, 0)$.

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52. $5x^2 - 2xy + 5y^2 - 12 = 0$

$$x + y - 1 = 0 \Rightarrow y = 1 - x$$
$$5x^2 - 2x(1 - x) + 5(1 - x)^2 - 12 = 0$$
$$5x^2 - 2x + 2x^2 + 5(1 - 2x + x^2) - 12 = 0$$
$$5x^2 - 2x + 2x^2 + 5 - 10x + 5x^2 - 12 = 0$$
$$12x^2 - 12x - 7 = 0$$
$$x = \frac{3 \pm \sqrt{30}}{6}$$

When $x = \dfrac{3 + \sqrt{30}}{6}$: $y = 1 - \dfrac{3 + \sqrt{30}}{6} = \dfrac{3 - \sqrt{30}}{6}$

When $x = \dfrac{3 - \sqrt{30}}{6}$: $y = 1 - \dfrac{3 - \sqrt{30}}{6} = \dfrac{3 + \sqrt{30}}{6}$

Points of intersection: $\left(\frac{1}{6}(3 + \sqrt{30}), \frac{1}{6}(3 - \sqrt{30})\right), \left(\frac{1}{6}(3 - \sqrt{30}), \frac{1}{6}(3 + \sqrt{30})\right)$

53. True. $B^2 - 4AC = 1 - 4k$

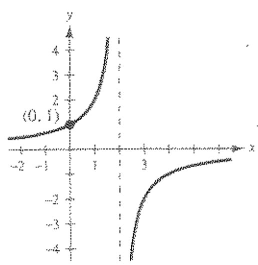
If $k < \frac{1}{4}$, then $B^2 - 4AC > 0$.

54. False. See Example 2. However, $A + C = A' + C'$.

55. $g(x) = \dfrac{2}{2 - x}$

Asymptotes:
$x = 2, y = 0$

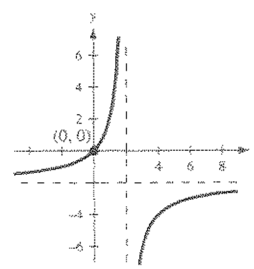
Intercepts: $(0, 1)$



56. $f(x) = \dfrac{2x}{2 - x} = -2 + \dfrac{4}{2 - x}$

Intercept: $(0, 0)$

Asymptotes:
$x = 2, y = -2$

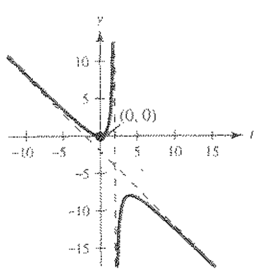


57. $h(t) = \dfrac{t^2}{2 - t} = -t - 2 + \dfrac{4}{2 - t}$

Slant asymptote: $y = -t - 2$

Vertical asymptote: $t = 2$

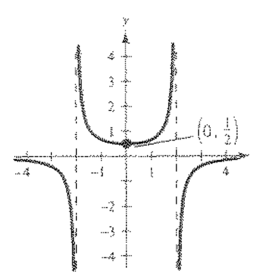
Intercept: $(0, 0)$



58. $g(s) = \dfrac{2}{4 - s^2}$

Intercept: $\left(0, \dfrac{1}{2}\right)$

Asymptotes:
$s = \pm 2, y = 0$



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59. (a) $AB = \begin{bmatrix} 1 & -3 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} 0 & 6 \\ 5 & -1 \end{bmatrix} = \begin{bmatrix} -15 & 9 \\ 25 & 7 \end{bmatrix}$

(b) $BA = \begin{bmatrix} 0 & 6 \\ 5 & -1 \end{bmatrix}\begin{bmatrix} 1 & -3 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 12 & 30 \\ 3 & -20 \end{bmatrix}$

(c) $A^2 = \begin{bmatrix} 1 & -3 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} 1 & -3 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} -5 & -18 \\ 12 & 19 \end{bmatrix}$

60. (a) $AB = \begin{bmatrix} 1 & 5 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ -3 & 8 \end{bmatrix} = \begin{bmatrix} -12 & 42 \\ 6 & -16 \end{bmatrix}$

(b) $BA = \begin{bmatrix} 3 & 2 \\ -3 & 8 \end{bmatrix}\begin{bmatrix} 1 & 5 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 3 & 11 \\ -3 & -31 \end{bmatrix}$

(c) $A^2 = \begin{bmatrix} 1 & 5 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 1 & 5 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 1 & -5 \\ 0 & 4 \end{bmatrix}$

61. (a) $AB = [4 \quad -2 \quad 5]\begin{bmatrix} 3 \\ -4 \\ 5 \end{bmatrix} = [12 + 8 + 25] = [45]$

(b) $BA = \begin{bmatrix} 3 \\ -4 \\ 5 \end{bmatrix}[4 \quad -2 \quad 5] = \begin{bmatrix} 12 & -6 & 15 \\ -16 & 8 & -20 \\ 20 & -10 & 25 \end{bmatrix}$

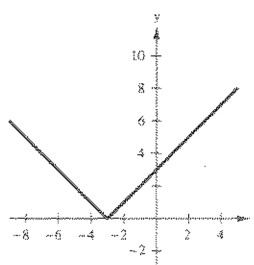
(c) A^2 does not exist.

62. (a) $AB = \begin{bmatrix} 8 & -10 & 2 \\ 27 & 20 & 6 \\ -13 & 20 & -13 \end{bmatrix}$

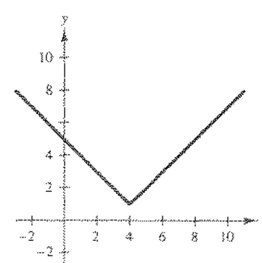
(b) $BA = \begin{bmatrix} -9 & -14 & 0 \\ 2 & 9 & 25 \\ 9 & -1 & 15 \end{bmatrix}$

(c) $A^2 = \begin{bmatrix} -2 & -2 & -10 \\ 16 & 19 & 5 \\ 4 & -2 & 20 \end{bmatrix}$

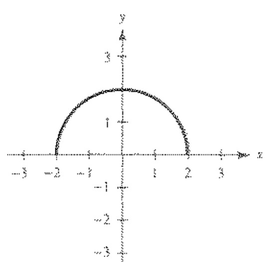
63. $f(x) = |x + 3|$



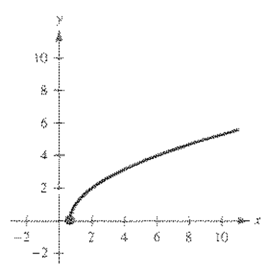
64. $f(x) = |x - 4| + 1$



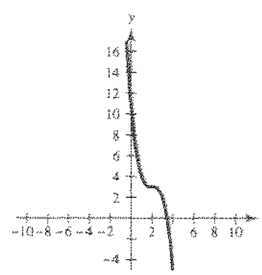
65. $g(x) = \sqrt{4 - x^2}$



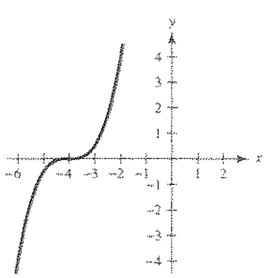
66. $g(x) = \sqrt{3x - 2}$



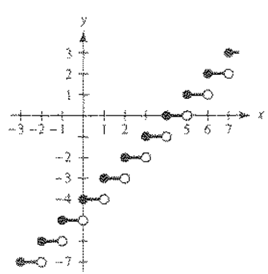
67. $h(t) = -(t - 2)^3 + 3$



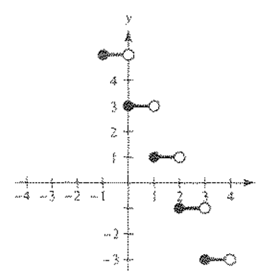
68. $h(t) = \frac{1}{2}(t + 4)^3$



69. $f(t) = [\![t - 5]\!] + 1$



70. $f(t) = -2[\![t]\!] + 3$



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71. Area $= \frac{1}{2}ab \sin C$

$= \frac{1}{2}(8)(12) \sin 110^\circ$

≈ 45.11

72. Area $= \frac{1}{2}ac \sin B$

$= \frac{1}{2}(25)(16) \sin 70^\circ$

≈ 187.94

73. $s = \frac{1}{2}(11 + 18 + 10) = \frac{39}{2}$

Area $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{\frac{39}{2}\left(\frac{17}{2}\right)\left(\frac{3}{2}\right)\left(\frac{19}{2}\right)}$

≈ 48.60

74. $s = \frac{1}{2}(23 + 35 + 27) = \frac{85}{2}$

Area $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{\frac{85}{2}\left(\frac{39}{2}\right)\left(\frac{15}{2}\right)\left(\frac{31}{2}\right)}$

≈ 310.39

Section 9.5 Parametric Equations

- If f and g are continuous functions of t on an interval I, then the set of ordered pairs $(f(t), g(t))$ is a *plane curve* C. The equations $x = f(t)$ and $y = g(t)$ are *parametric equations* for C and t is the *parameter*.
- You should be able to graph plane curves with your graphing utility.
- To eliminate the parameter:
  Solve for t in one equation and substitute into the second equation.
- You should be able to find the parametric equations for a graph.

Vocabulary Check

1. plane curve, parametric equations, parameter
2. orientation
3. eliminating, parameter

1. $x = t$

$y = t + 2$

$y = x + 2$, line

Matches (c).

2. $x = t^2$

$y = t - 2 \implies t = y + 2$

$x = (y + 2)^2$

Parabola opening to the right
Matches (d).

3. $x = \sqrt{t}$

$y = t$

$y = x^2$, parabola, $x \geq 0$

Matches (b).

4. $x = \dfrac{1}{t} \implies t = \dfrac{1}{x}$

$y = t + 2 \implies y = \dfrac{1}{x} + 2$

Matches (a).

5. $x = \ln t \iff t = e^x$

$y = \dfrac{1}{2}t - 2$

$y = \dfrac{1}{2}e^x - 2$

Matches (f).

6. $x = -2\sqrt{t} \implies t = \left(\dfrac{x}{-2}\right)^2 = \dfrac{x^2}{4}$

$y = e^t$

$y = e^{x^2/4}$

Exponential curve on $x \leq 0$

Matches (e).

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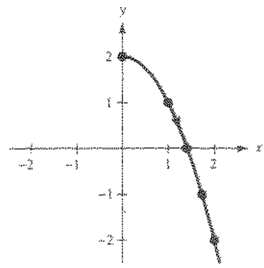
7. $x = \sqrt{t}, y = 2 - t$

(a)

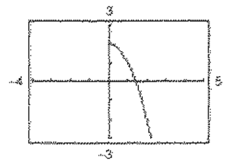
t	0	1	2	3	4
x	0	1	$\sqrt{2}$	$\sqrt{3}$	2
y	2	1	0	-1	-2

(b) Graph by hand.

Note: $x \geq 0$



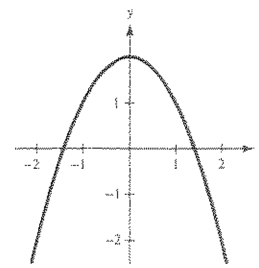
(c)



(d) $y = 2 - t = 2 - x^2$,

Parabola

In part (c), $x \geq 0$.

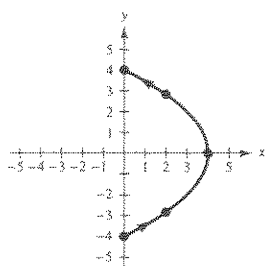


8. $x = 4\cos^2\theta, y = 4\sin\theta$

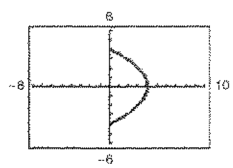
(a)

θ	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$
x	0	2	4	2	0
y	-4	$-2\sqrt{2}$	0	$2\sqrt{2}$	4

(b)



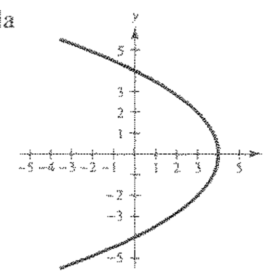
(c)



(d) $4x + y^2 = 16\cos^2\theta + 16\sin^2\theta = 16$

$\frac{x}{4} + \frac{y^2}{16} = 1$, parabola

The graph is an entire parabola rather than just the right portion.

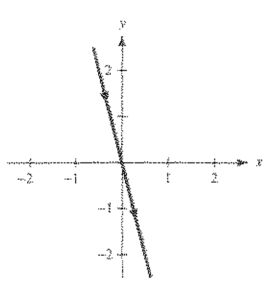


9. The graph opens upward, contains $(1, 0)$, and is oriented left to right. Matches (b).

10. The orientation of the graph is clockwise and the center is $(2, 3)$. Matches (c).

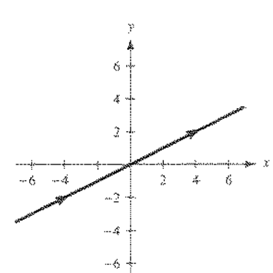
11. $x = t, y = -4t$

$y = -4x$



12. $x = t,\ y = \frac{1}{2}t$

$y = \frac{1}{2}x$ or $x - 2y = 0$



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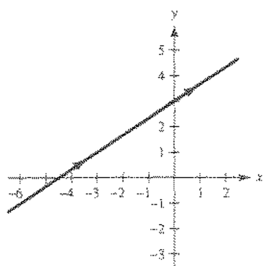
13. $x = 3t - 3, y = 2t + 1$

$$t = \frac{x+3}{3}$$

$$y = 2\left(\frac{x+3}{3}\right) + 1$$

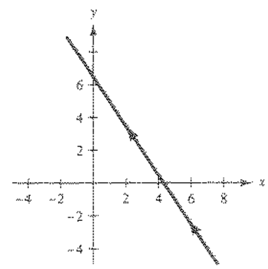
$$y = \frac{2}{3}x + 3$$



14. $x = 3 - 2t, y = 2 + 3t$

$$y = 2 + 3\left(\frac{3-x}{2}\right)$$

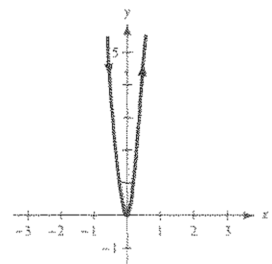
$$3x + 2y - 13 = 0$$



15. $x = \frac{1}{4}t, y = t^2$

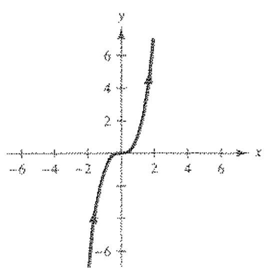
$$y = (4x)^2$$

$$y = 16x^2$$



16. $x = t,\ y = t^3$

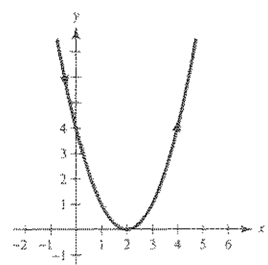
$$y = x^3$$



17. $x = t + 2, y = t^2$

$$t = x - 2$$

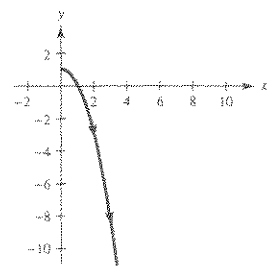
$$y = (x-2)^2$$



18. $x = \sqrt{t}$

$$y = 1 - t$$

$$y = 1 - x^2,\ x \ge 0$$

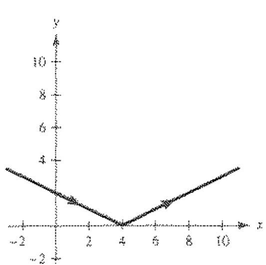


19. $x = 2t, y = |t - 2|$

$$t = \frac{x}{2} \implies y = |t - 2|$$

$$= \left|\frac{x}{2} - 2\right|$$

$$= \frac{1}{2}|x - 4|$$



20. $x = |t - 1|$

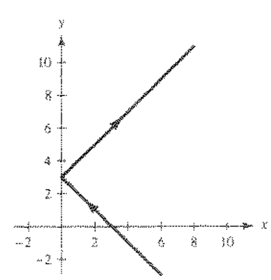
$$y = t + 2$$

Eliminating the parameter t, $t = y - 2$ and

$$x = |t - 1|$$

$$= |(y - 2) - 1|$$

$$= |y - 3|.$$



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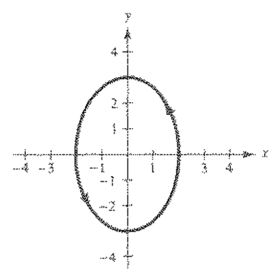
21. $x = 2\cos\theta,\ y = 3\sin\theta$

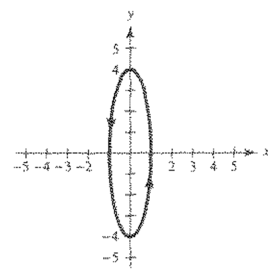


$$\left(\frac{x}{2}\right)^2 = \cos^2\theta, \left(\frac{y}{3}\right)^2 = \sin^2\theta$$

$$\frac{x^2}{4} + \frac{y^2}{9} = \cos^2\theta + \sin^2\theta = 1$$

$$\frac{x^2}{4} + \frac{y^2}{9} = 1, \quad \text{ellipse}$$

22. $x = \cos\theta,\ y = 4\sin\theta$



$$x^2 + \left(\frac{y}{4}\right)^2 = \cos^2\theta + \sin^2\theta = 1$$

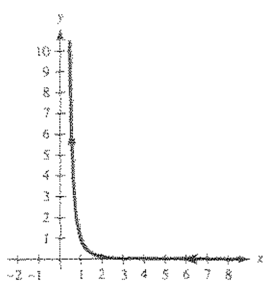
$$x^2 + \frac{y^2}{16} = 1, \quad \text{ellipse}$$

23. $x = e^{-t} \implies \dfrac{1}{x} = e^t$

$$y = e^{3t} \implies y = (e^t)^3$$

$$y = \left(\frac{1}{x}\right)^3$$

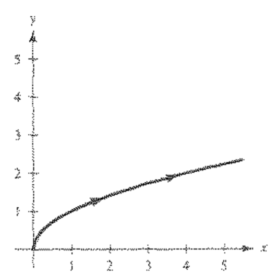
$$y = \frac{1}{x^3},\ x > 0,\ y > 0$$



24. $x = e^{2t}$

$$y = e^t \implies y^2 = e^{2t}$$

$$y^2 = x,\ y > 0;\ y = \sqrt{x},\ x > 0$$

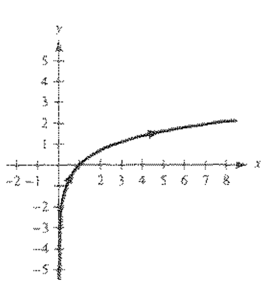


25. $x = t^3 \implies x^{1/3} = t$

$$y = 3\ln t \implies y = \ln t^3$$

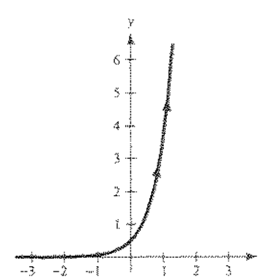
$$y = \ln(x^{1/3})^3$$

$$y = \ln x$$



26. $x = \ln 2t \implies e^x = 2t \implies t = \frac{1}{2}e^x$

$$y = 2t^2 = 2\left(\tfrac{1}{2}e^x\right)^2 = \tfrac{1}{2}e^{2x}$$



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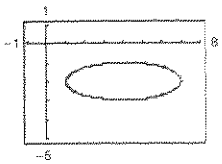
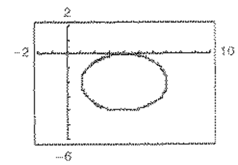
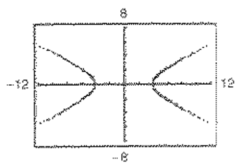
27. $x = 4 + 3\cos\theta,\ y = -2 + \sin\theta$



28. $x = 4 + 3\cos\theta,\ y = -2 + 2\sin\theta$

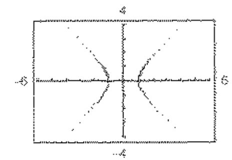


29. $x = 4\sec\theta,\ y = 2\tan\theta$



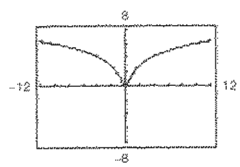
30. $x = \sec\theta$

$y = \tan\theta$



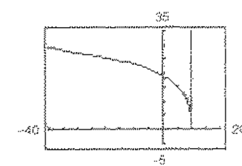
31. $x = \dfrac{t}{2}$

$y = \ln(t^2 + 1)$



32. $x = 10 - 0.01e^t$

$y = 0.4t^2$



33. By eliminating the parameters in (a)–(d), we get $y = 2x + 1$. They differ from each other in restricted domain and in orientation.

(a) Domain: $-\infty < x < \infty$
Orientation: Left to right

(b) Domain: $-1 \le x \le 1$
Orientation: Depends on θ

(c) Domain: $0 < x < \infty$
Orientation: Right to left

(d) Domain: $0 < x < \infty$
Orientation: Left to right

34. Each curve represents a portion of the line $2y + x - 8 = 0$.

(a) $x = 2\sqrt{t},\ x \ge 0$

$$y = 4 - \sqrt{t} = 4 - \frac{x}{2},\ y \le 4$$

Orientation: Left to right

(b) $x = 2\sqrt[3]{t},\ -\infty < x < \infty$

$$y = 4 - \sqrt[3]{t} = 4 - \frac{x}{2}$$

Orientation: Left to right

(c) $x = 2(t + 1),\ -\infty < x < \infty$

$$y = 3 - t = 3 - \left(\frac{x - 2}{2}\right) = 4 - \frac{x}{2}$$

Orientation: Left to right

(d) $x = -2t^2,\ x \le 0$

$$y = 4 + t^2 = 4 - \frac{x}{2}$$

Orientation: Left to right for $t \le 0$
Right to left for $t > 0$

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35. $t = \dfrac{(x - x_1)}{(x_2 - x_1)}$

$y = y_1 + \left(\dfrac{x - x_1}{x_2 - x_1}\right)(y_2 - y_1)$

$\Longrightarrow y - y_1 = \left(\dfrac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$

36. $x = h + r\cos\theta$

$y = k + r\sin\theta$

$\dfrac{(x - h)}{r} = \cos\theta, \ \dfrac{y - k}{r} = \sin\theta$

$\cos^2\theta + \sin^2\theta = \dfrac{(x - h)^2}{r^2} + \dfrac{(y - k)^2}{r^2} = 1$

$(x - h)^2 + (y - k)^2 = r^2$

37. $x = h + a\cos\theta$

$y = k + b\sin\theta$

$\dfrac{x - h}{a} = \cos\theta, \ \dfrac{y - k}{b} = \sin\theta$

$\dfrac{(x - h)^2}{a^2} + \dfrac{(y - k)^2}{b^2} = 1$

38. $x = h + a\sec\theta$

$y = k + b\tan\theta$

$\dfrac{x - h}{a} = \sec\theta, \ \dfrac{y - k}{b} = \tan\theta$

$\sec^2\theta - \tan^2\theta = \dfrac{(x - h)^2}{a^2} - \dfrac{(y - k)^2}{b^2} = 1$

39. $x = x_1 + t(x_2 - x_1) = 1 + t(6 - 1) = 1 + 5t$

$y = y_1 + t(y_2 - y_1) = 4 + t(-3 - 4) = 4 - 7t$

40. $x = h + r\cos\theta = 2 + 4\cos\theta$

$y = k + r\sin\theta = 5 + 4\sin\theta$

41. $a = 5, c = 4$, and $b = \sqrt{a^2 - c^2} = 3$.

The center is $(0, 0)$, so $h = 0$ and $k = 0$.

$\cos^2\theta + \sin^2\theta = 1 = \dfrac{x^2}{5^2} + \dfrac{y^2}{3^2}$, so $x = 5\cos\theta$ and $y = 3\sin\theta$. This solution is not unique.

42. $a = 1, c = 2$, and $b = \sqrt{c^2 - a^2} = \sqrt{3}$.

The center is $(0, 0)$, so $h = 0$ and $k = 0$.

$\sec^2\theta - \tan^2\theta = 1 = \dfrac{y^2}{1} - \dfrac{x^2}{3}$, so $y = \sec\theta$ and $x = \sqrt{3}\tan\theta$.

43. $y = 5x - 3$

Answers will vary.

$x = t, y = 5t - 3$

$x = \frac{1}{5}t, y = t - 3$

44. $y = 4 - 7x$

Answers will vary.

$x = t, y = 4 - 7t$

$x = 2t, y = 4 - 14t$

45. $y = \dfrac{1}{x}$

Sample answers:

$x = t, y = \dfrac{1}{t}$

$x = t^3, y = \dfrac{1}{t^3}$

46. $y = \dfrac{1}{2x}$

Sample answers:

$x = t, y = \dfrac{1}{2t}$

$x = 2t, y = \dfrac{1}{4t}$

47. $y = 6x^2 - 5$

Sample answers:

$x = t, y = 6t^2 - 5$

$x = 2t, y = 24t^2 - 5$

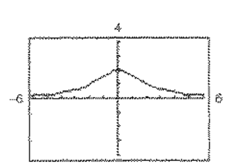
48. $y = x^3 + 2x$

Sample answers:

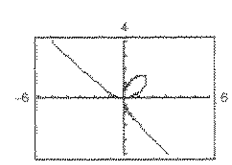
$x = t, y = t^3 + 2t$

$x = \frac{1}{2}t, y = \dfrac{t^3}{8} + t$

49. $x = 2\cot\theta, \ y = 2\sin^2\theta$



50. $x = \dfrac{3t}{1 + t^3}, \ y = \dfrac{3t^2}{1 + t^3}$



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51. Matches (b).

52. Matches (c).

53. Matches (d).

54. Matches (a).

55. $x = (v_0 \cos \theta)t$, $y = h + (v_0 \sin \theta)t - 16t^2$

(a) $100 \text{ miles/hour} = \dfrac{100 \text{ mi/hr} \cdot 5280 \text{ ft/mi}}{3600 \text{ sec/hr}}$

$= 146.67 \text{ ft/sec}$

$x = (146.67 \cos \theta)t$

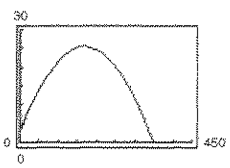
$y = 3 + (146.67 \sin \theta)t - 16t^2$

(b) $\theta = 15°$

$x = (146.67 \cos 15°)t = 141.7t$

$y = 3 + (146.67 \sin 15°)t - 16t^2$

$= 3 + 38.0t - 16t^2$



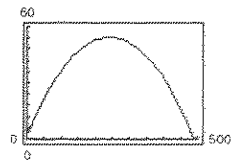
It is not a home run because $y < 10$ when $x = 400$.

(c) $\theta = 23°$

$x = (146.67 \cos 23°)t = 135.0t$

$y = 3 + (146.67 \sin 23°)t - 16t^2$

$= 3 + 57.3t - 16t^2$



Yes, it is a home run because $y > 10$ when $x = 400$.

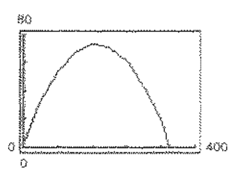
(d) $\theta \approx 19.4°$ is the minimum angle.

56. (a) $x = (v_0 \cos \theta)t = (105 \cos 40°)t$

$y = h + (v_0 \sin \theta)t - 16t^2$

$= 2.5 + (105 \sin 40°)t - 16t^2$

(b)



The maximum height is approximately 73.68 feet, when $t \approx 2.109$ seconds.

(c) The horizontal distance is approximately 342.25 feet.

(d) You could use the Quadratic Formula to find the zeros of $y = -16t^2 + (105 \sin 40°)t + 2.5$. The larger zero, 4.255, gives $x \approx 342.25$ feet.

57. (a) $x = (\cos 35°)v_0 t$

$y = 7 + (\sin 35°)v_0 t - 16t^2$

(b) If the ball is caught at time t_1, then:

$90 = (\cos 35°)v_0 t_1$

$4 = 7 + (\sin 35°)v_0 t_1 - 16t_1^{\,2}$.

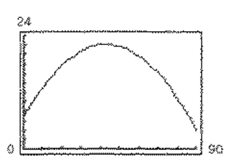
$v_0 t_1 = \dfrac{90}{\cos 35°} \implies -3 = (\sin 35°)\dfrac{90}{\cos 35°} - 16t_1^{\,2}$

$\implies 16t_1^{\,2} = 90 \tan 35° + 3$

$\implies t_1 \approx 2.03$ seconds

$\implies v_0 = \dfrac{90}{t_1 \cos 35°} \approx 54.09$ ft/sec

(c)



Maximum height ≈ 22 feet

(d) From part (b), $t_1 \approx 2.03$ seconds.

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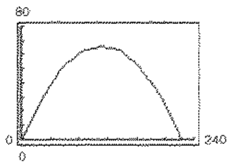
58. (a) $x = (v_0 \cos \theta)t = (85 \cos 50°)t$

$y = h + (v_0 \sin \theta)t - 16t^2 = (85 \sin 50°)t - 16t^2$

(b) 

The maximum height is approximately 66.25 feet when $t \approx 2.035$ seconds.

(c) The horizontal distance is approximately 222.35 feet.

(d) You could solve the equation $y = (85 \sin 50°)t - 16t^2 = 0$ for $t \approx 4.0696$. Then, $x \approx 222.35$ feet.

59. True

$x = t$ first set

$y = t^2 + 1 = x^2 + 1$

$x = 3t$ second set

$y = 9t^2 + 1 = (3t)^2 + 1 = x^2 + 1$

60. False. The graph of $x = t^2, y = t^2$ represents the portion of the line $y = x$ in the first quadrant.

61. False. For example, $x = t^2$ and $y = t$ does not represent y as a function of x.

62. False. The equations represent a line.

63. Sample answer: $x = \cos \theta$

$y = -2 \sin \theta$

64. The graph is the same, but the orientation is reversed.

65. $f(-x) = \dfrac{4(-x)^2}{(-x)^2 + 1} = \dfrac{4x^2}{x^2 + 1} = f(x)$

Symmetric about the y-axis

Even function

66. $f(x) = \sqrt{x}, x \geq 0$

No symmetry

Neither even nor odd

67. $y = e^x \neq e^{-x}; e^{-x} \neq -e^x$

No symmetry

Neither even nor odd

68. $(x - 2)^2 = y + 4$

$y = x^2 - 4x$

No symmetry

Neither even nor odd

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Section 9.6 Polar Coordinates

- In polar coordinates you do not have unique representation of points. The point (r, θ) can be represented by $(r, \theta \pm 2n\pi)$ or by $(-r, \theta \pm (2n + 1)\pi)$ where n is any integer. The pole is represented by $(0, \theta)$ where θ is any angle.
- To convert from polar coordinates to rectangular coordinates, use the following relationships.

  $x = r \cos \theta$

  $y = r \sin \theta$
- To convert from rectangular coordinates to polar coordinates, use the following relationships.

  $r = \pm\sqrt{x^2 + y^2}$

  $\tan \theta = y/x$

  If θ is in the same quadrant as the point (x, y), then r is positive. If θ is in the opposite quadrant as the point (x, y), then r is negative.
- You should be able to convert rectangular equations to polar form and vice versa.

Vocabulary Check

1. pole **2.** directed distance, directed angle **3.** polar

1. Polar coordinates: $\left(4, \dfrac{\pi}{2}\right)$

$x = 4 \cos\left(\dfrac{\pi}{2}\right) = 0$

$y = 4 \sin\left(\dfrac{\pi}{2}\right) = 4$

Rectangular coordinates: $(0, 4)$

2. Polar coordinates: $\left(4, \dfrac{3\pi}{2}\right)$

$x = 4 \cos\left(\dfrac{3\pi}{2}\right) = 0, y = 4 \sin\left(\dfrac{3\pi}{2}\right) = -4$

Rectangular coordinates: $(0, -4)$

3. Polar coordinates: $\left(-1, \dfrac{5\pi}{4}\right)$

$x = -1 \cos\left(\dfrac{5\pi}{4}\right) = \dfrac{\sqrt{2}}{2}$

$y = -1 \sin\left(\dfrac{5\pi}{4}\right) = \dfrac{\sqrt{2}}{2}$

Rectangular coordinates: $\left(\dfrac{\sqrt{2}}{2}, \dfrac{\sqrt{2}}{2}\right)$

4. Polar coordinates: $\left(2, -\dfrac{\pi}{4}\right)$

$x = 2 \cos\left(-\dfrac{\pi}{4}\right) = 2\left(\dfrac{\sqrt{2}}{2}\right) = \sqrt{2}$

$y = 2 \sin\left(-\dfrac{\pi}{4}\right) = 2\left(-\dfrac{\sqrt{2}}{2}\right) = -\sqrt{2}$

Rectangular coordinates: $(\sqrt{2}, -\sqrt{2})$

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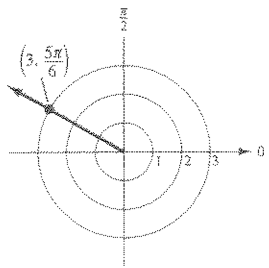
5.



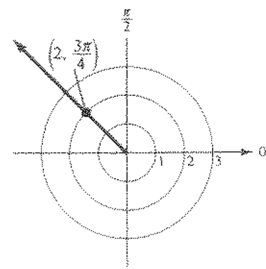
Three additional representations:

$$\left(3, \frac{5\pi}{6} - 2\pi\right) = \left(3, -\frac{7\pi}{6}\right)$$

$$\left(-3, \frac{5\pi}{6} + \pi\right) = \left(-3, \frac{11\pi}{6}\right)$$

$$\left(-3, \frac{5\pi}{6} - \pi\right) = \left(-3, -\frac{\pi}{6}\right)$$

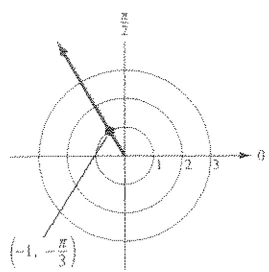
6.



Three additional points:

$$\left(2, \frac{-5\pi}{4}\right), \left(-2, \frac{7\pi}{4}\right), \left(-2, \frac{-\pi}{4}\right)$$

7.



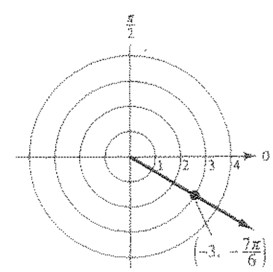
Three additional representations:

$$\left(-1, -\frac{\pi}{3} + 2\pi\right) = \left(-1, \frac{5\pi}{3}\right)$$

$$\left(1, -\frac{\pi}{3} + \pi\right) = \left(1, \frac{2\pi}{3}\right)$$

$$\left(1, -\frac{\pi}{3} - \pi\right) = \left(1, -\frac{4\pi}{3}\right)$$

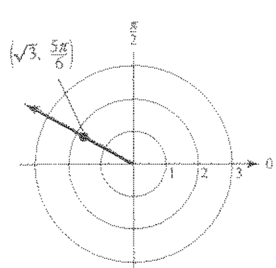
8.



Three additional points:

$$\left(-3, \frac{5\pi}{6}\right), \left(3, \frac{11\pi}{6}\right), \left(3, -\frac{\pi}{6}\right)$$

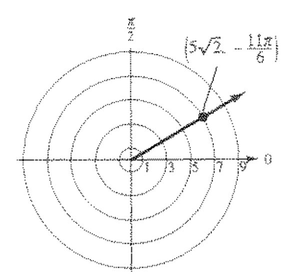
9.



Three additional representations:

$$\left(\sqrt{3}, -\frac{7\pi}{6}\right), \left(-\sqrt{3}, -\frac{\pi}{6}\right), \left(-\sqrt{3}, \frac{11\pi}{6}\right)$$

10.



Three additional points:

$$\left(5\sqrt{2}, \frac{\pi}{6}\right), \left(-5\sqrt{2}, -\frac{5\pi}{6}\right), \left(-5\sqrt{2}, \frac{7\pi}{6}\right)$$

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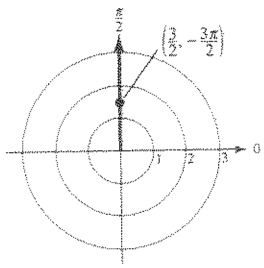
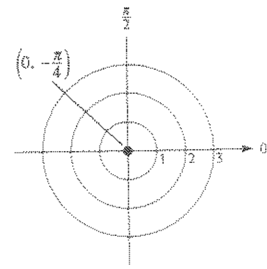
11.



Three additional representations:

$\left(\frac{3}{2}, \frac{\pi}{2}\right), \left(-\frac{3}{2}, \frac{3\pi}{2}\right), \left(-\frac{3}{2}, -\frac{\pi}{2}\right)$

12.



$\left(0, \frac{-\pi}{4}\right)$ is the origin.

Three additional points: $\left(0, \frac{3\pi}{4}\right), \left(0, \frac{-5\pi}{4}\right), \left(0, \frac{7\pi}{4}\right)$

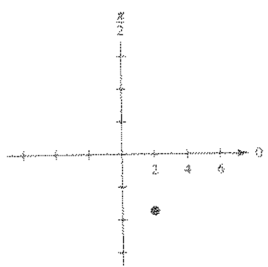
(Any angle will do since $r = 0$.)

13. Polar coordinates: $\left(4, -\frac{\pi}{3}\right)$

$$x = 4\cos\left(-\frac{\pi}{3}\right) = 2$$

$$y = 4\sin\left(-\frac{\pi}{3}\right) = -2\sqrt{3}$$

Rectangular coordinates: $\left(2, -2\sqrt{3}\right)$

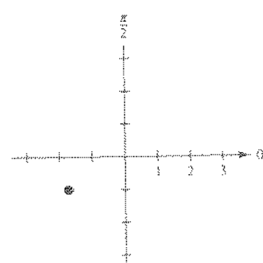


14. Polar coordinates: $\left(2, \frac{7\pi}{6}\right)$

$$x = 2\cos\frac{7\pi}{6} = 2\left(-\frac{\sqrt{3}}{2}\right) = -\sqrt{3}$$

$$y = 2\sin\frac{7\pi}{6} = 2\left(-\frac{1}{2}\right) = -1$$

Rectangular coordinates: $\left(-\sqrt{3}, -1\right)$

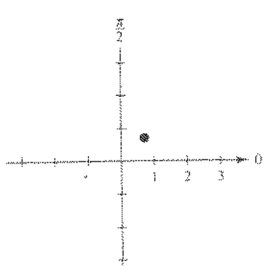


15. Polar coordinates: $\left(-1, \frac{-3\pi}{4}\right)$

$$x = -1\cos\left(\frac{-3\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

$$y = -1\sin\left(\frac{-3\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

Rectangular coordinates: $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

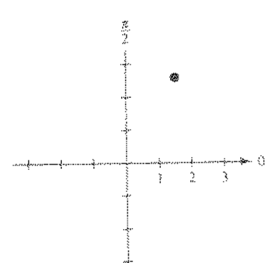


16. Polar coordinates: $\left(-3, -\frac{2\pi}{3}\right) = \left(3, \frac{\pi}{3}\right)$

$$x = -3\cos\left(-\frac{2\pi}{3}\right) = -3\left(-\frac{1}{2}\right) = \frac{3}{2}$$

$$y = -3\sin\left(-\frac{2\pi}{3}\right) = -3\left(-\frac{\sqrt{3}}{2}\right) = \frac{3\sqrt{3}}{2}$$

Rectangular coordinates: $\left(\frac{3}{2}, \frac{3\sqrt{3}}{2}\right)$



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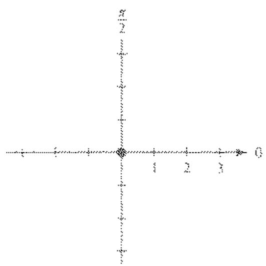
17. Polar coordinates: $\left(0, -\dfrac{7\pi}{6}\right)$ (origin!)

$x = 0\cos\left(-\dfrac{7\pi}{6}\right) = 0$

$y = 0\sin\left(-\dfrac{7\pi}{6}\right) = 0$

Rectangular coordinates: $(0, 0)$

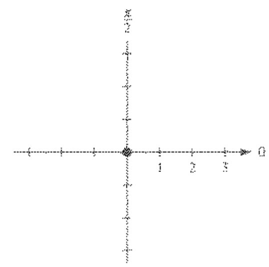


18. Polar coordinates: $\left(0, \dfrac{5\pi}{4}\right)$ (origin!)

$x = 0\cos\dfrac{5\pi}{4} = 0$

$y = 0\sin\dfrac{5\pi}{4} = 0$

Rectangular coordinates: $(0, 0)$

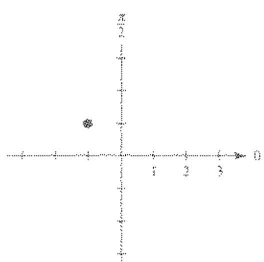


19. Polar coordinates: $(\sqrt{2}, 2.36)$

$x = \sqrt{2}\cos(2.36) \approx -1.004$

$y = \sqrt{2}\sin(2.36) \approx 0.996$

Rectangular coordinates: $(-1.004, 0.996)$

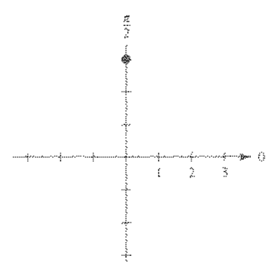


20. Polar coordinates: $(-3, -1.57)$

$x = -3\cos(-1.57) \approx -0.0024$

$y = -3\sin(-1.57) \approx 3.000$

Rectangular coordinates: $(-0.0024, 3)$



21. $(r, \theta) = \left(2, \dfrac{2\pi}{9}\right) \implies (x, y) = (1.53, 1.29)$

22. $(r, \theta) = \left(4, \dfrac{11\pi}{9}\right) \implies (x, y) = (-3.06, -2.57)$

23. $(r, \theta) = (-4.5, 1.3) \implies (x, y) = (-1.204, -4.336)$

24. $(r, \theta) = (8.25, 3.5) \implies (x, y) = (-7.726, -2.894)$

25. $(r, \theta) = (2.5, 1.58) \implies (x, y) = (-0.02, 2.50)$

26. $(r, \theta) = (5.4, 2.85) \implies (x, y) = (-5.17, 1.55)$

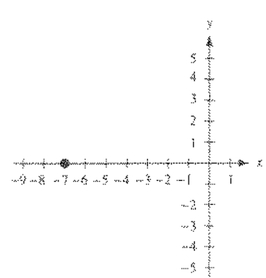
27. $(r, \theta) = (-4.1, -0.5) \implies (x, y) = (-3.60, 1.97)$

28. $(r, \theta) = (8.2, -3.2) \implies (x, y) = (-8.19, 0.48)$

29. Rectangular coordinates: $(-7, 0)$

$r = 7$, $\tan\theta = 0$, $\theta = 0$

Polar coordinates: $(7, \pi)$, $(-7, 0)$



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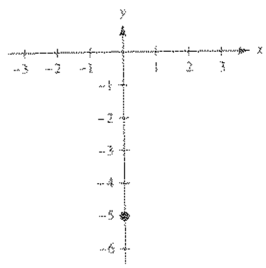
30. Rectangular coordinates: $(0, -5)$

$r = 5$, $\tan\theta$ undefined, $\theta = \dfrac{\pi}{2}$

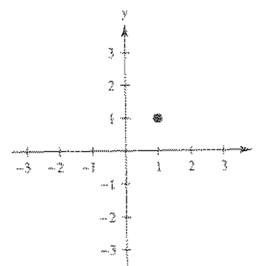
Polar coordinates: $\left(5, \dfrac{3\pi}{2}\right), \left(-5, \dfrac{\pi}{2}\right)$



31. Rectangular coordinates: $(1, 1)$

$r = \sqrt{2}$, $\tan\theta = 1$, $\theta = \dfrac{\pi}{4}$

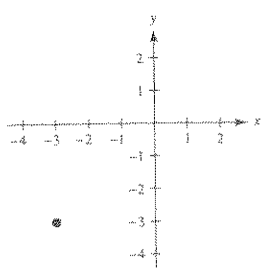
Polar coordinates: $\left(\sqrt{2}, \dfrac{\pi}{4}\right), \left(-\sqrt{2}, \dfrac{5\pi}{4}\right)$



32. Rectangular coordinates: $(-3, -3)$

$r = 3\sqrt{2}$, $\tan\theta = 1$, $\theta = \dfrac{\pi}{4}$

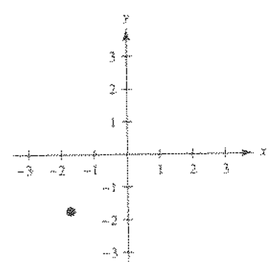
Polar coordinates: $\left(3\sqrt{2}, \dfrac{5\pi}{4}\right), \left(-3\sqrt{2}, \dfrac{\pi}{4}\right)$



33. Rectangular coordinates: $\left(-\sqrt{3}, -\sqrt{3}\right)$

$r = \sqrt{3 + 3} = \sqrt{6}$, $\tan\theta = 1$, $\theta = \dfrac{\pi}{4}$

Polar coordinates: $\left(\sqrt{6}, \dfrac{5\pi}{4}\right), \left(-\sqrt{6}, \dfrac{\pi}{4}\right)$

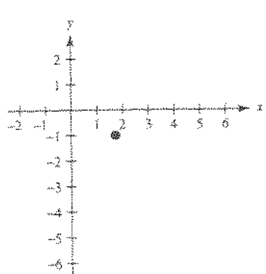


34. Rectangular coordinates: $\left(\sqrt{3}, -1\right)$

$r = \sqrt{3 + 1} = 2$

$\tan\theta = \dfrac{-1}{\sqrt{3}}$, $\theta = \dfrac{11\pi}{6}$

Polar coordinates: $\left(2, \dfrac{11\pi}{6}\right), \left(-2, \dfrac{5\pi}{6}\right)$

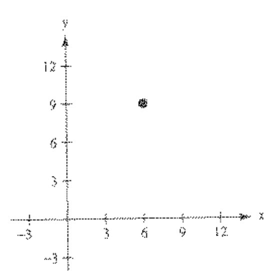


35. $(x, y) = (6, 9)$

$r = \sqrt{6^2 + 9^2} = \sqrt{117} \approx 10.8$

$\tan\theta = \dfrac{9}{6} = \dfrac{3}{2} \implies \theta \approx 0.983$

Polar coordinates: $(10.8, 0.983), (-10.8, 4.124)$



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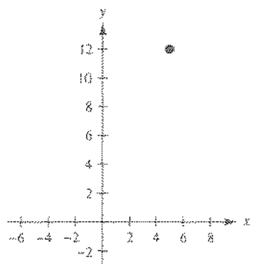
36. Rectangular coordinates: $(5, 12)$

$r = \sqrt{25 + 144} = 13$, $\tan\theta = \frac{12}{5}$, $\theta \approx 1.176$

Polar coordinates: $(13, 1.176)$, $(-13, 4.318)$



37. $(x, y) = (3, -2) \implies r = \sqrt{3^2 + (-2)^2} = \sqrt{13}$

$\theta = \arctan\left(-\frac{2}{3}\right) \approx -0.588$

$(r, \theta) \approx \left(\sqrt{13}, -0.588\right)$

38. $(x, y) = (-5, 2) \implies (r, \theta) = (5.39, 2.76)$

39. $(x, y) = \left(\sqrt{3}, 2\right) \implies r = \sqrt{3 + 2^2} = \sqrt{7}$

$\theta = \arctan\left(\dfrac{2}{\sqrt{3}}\right) \approx 0.857$

$(r, \theta) \approx \left(\sqrt{7}, 0.857\right)$

40. $(x, y) = \left(3\sqrt{2}, 3\sqrt{2}\right) \implies (r, \theta) = \left(6, \dfrac{\pi}{4}\right) \approx (6.0, 0.785)$

41. $(x, y) = \left(\dfrac{5}{2}, \dfrac{4}{3}\right) \implies r = \sqrt{\left(\dfrac{5}{2}\right)^2 + \left(\dfrac{4}{3}\right)^2} = \dfrac{17}{6}$

$\theta = \arctan\left(\dfrac{4/3}{5/2}\right) \approx 0.490$

$(r, \theta) \approx \left(\dfrac{17}{6}, 0.490\right)$

42. $(x, y) = \left(\dfrac{7}{4}, \dfrac{3}{2}\right) \implies (r, \theta) = (2.30, 0.71)$

43. $x^2 + y^2 = 9$

$r^2 = 9$

$r = 3$

44. $x^2 + y^2 = 16$

$r^2 = 16$

$r = 4$

45. $y = 4$

$r\sin\theta = 4$

$r = 4\csc\theta$

46. $y = x$

$r\sin\theta = r\cos\theta$

$\sin\theta = \cos\theta$

$\tan\theta = 1$

$\theta = \dfrac{\pi}{4}$

47. $x = 8$

$r\cos\theta = 8$

$r = 8\sec\theta$

48. $x = a$

$r\cos\theta = a$

$r = a\sec\theta$

49. $3x - 6y + 2 = 0$

$3r\cos\theta - 6r\sin\theta = -2$

$r(3\cos\theta - 6\sin\theta) = -2$

$r = \dfrac{2}{6\sin\theta - 3\cos\theta}$

50. $4x + 7y - 2 = 0$

$4r\cos\theta + 7r\sin\theta - 2 = 0$

$r(4\cos\theta + 7\sin\theta) = 2$

$r = \dfrac{2}{4\cos\theta + 7\sin\theta}$

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51. $$\begin{aligned} xy &= 4 \\ (r\cos\theta)(r\sin\theta) &= 4 \\ r^2\cos\theta\sin\theta &= 4 \\ r^2(2\cos\theta\sin\theta) &= 8 \\ r^2\sin 2\theta &= 8 \\ r^2 &= 8\csc 2\theta \end{aligned}$$

52. $$\begin{aligned} 2xy &= 1 \\ 2r\cos\theta \cdot r\sin\theta &= 1 \\ 2r^2 &= \sec\theta\csc\theta \\ r^2 &= \tfrac{1}{2}\sec\theta\csc\theta = \csc(2\theta) \end{aligned}$$

53. $$\begin{aligned} (x^2+y^2)^2 &= 9(x^2-y^2) \\ (r^2)^2 &= 9(r^2\cos^2\theta - r^2\sin^2\theta) \\ r^2 &= 9(\cos^2\theta - \sin^2\theta) \\ r^2 &= 9\cos(2\theta) \end{aligned}$$

54. $$\begin{aligned} y^2 - 8x - 16 &= 0 \\ r^2\sin^2\theta - 8r\cos\theta - 16 &= 0 \\ r^2(1-\cos^2\theta) - 8r\cos\theta - 16 &= 0 \\ r^2\cos^2\theta + 8r\cos\theta + 16 &= r^2 \\ (r\cos\theta + 4)^2 &= r^2 \\ r &= \pm(r\cos\theta + 4) \end{aligned}$$

$$r = \frac{4}{1-\cos\theta} \quad \text{or} \quad r = \frac{-4}{1+\cos\theta}$$

55. $$\begin{aligned} x^2+y^2-6x &= 0 \\ r^2 - 6r\cos\theta &= 0 \\ r^2 &= 6r\cos\theta \\ r &= 6\cos\theta \end{aligned}$$

56. $$\begin{aligned} x^2+y^2-8y &= 0 \\ r^2 - 8r\sin\theta &= 0 \\ r(r - 8\sin\theta) &= 0 \\ r &= 8\sin\theta \end{aligned}$$

57. $$\begin{aligned} x^2+y^2-2ax &= 0 \\ r^2 - 2ar\cos\theta &= 0 \\ r(r - 2a\cos\theta) &= 0 \\ r &= 2a\cos\theta \end{aligned}$$

58. $$\begin{aligned} x^2+y^2-2ay &= 0 \\ r^2 - 2a\,r\sin\theta &= 0 \\ r(r - 2a\sin\theta) &= 0 \\ r &= 2a\sin\theta \end{aligned}$$

59. $$\begin{aligned} y^2 &= x^3 \\ (r\sin\theta)^2 &= (r\cos\theta)^3 \\ \sin^2\theta &= r\cos^3\theta \\ r &= \frac{\sin^2\theta}{\cos^3\theta} \\ &= \tan^2\theta\sec\theta \end{aligned}$$

60. $$\begin{aligned} x^2 &= y^3 \\ r^2\cos^2\theta &= r^3\sin^3\theta \\ r &= \frac{\cos^2\theta}{\sin^3\theta} = \cot^2\theta\csc\theta \end{aligned}$$

61. $$\begin{aligned} r &= 6\sin\theta \\ r^2 &= 6r\sin\theta \\ x^2+y^2 &= 6y \\ x^2+y^2-6y &= 0 \end{aligned}$$

62. $$\begin{aligned} r &= 2\cos\theta \\ r^2 &= 2r\cos\theta \\ x^2+y^2 &= 2x \end{aligned}$$

63. $$\begin{aligned} \theta &= \frac{4\pi}{3} \\ \tan\theta &= \tan\frac{4\pi}{3} = \frac{y}{x} \\ \sqrt{3} &= \frac{y}{x} \\ y &= \sqrt{3}x \end{aligned}$$

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64. $$\theta = \frac{5\pi}{3}$$
$$\tan\theta = \tan\frac{5\pi}{3} = -\sqrt{3}$$
$$\frac{y}{x} = -\sqrt{3}$$
$$y + \sqrt{3}x = 0$$

65. $$\theta = \frac{5\pi}{6}$$
$$\tan\theta = \tan\frac{5\pi}{6} = \frac{y}{x}$$
$$\frac{-\sqrt{3}}{3} = \frac{y}{x}$$
$$y = \frac{-\sqrt{3}}{3}x$$

66. $$\theta = \frac{11\pi}{6}$$
$$\tan\theta = \tan\frac{11\pi}{6} = \frac{y}{x}$$
$$\frac{-\sqrt{3}}{3} = \frac{y}{x}$$
$$y = \frac{-\sqrt{3}}{3}x$$

67. $\theta = \frac{\pi}{2}$, vertical line
$$x = 0$$

68. $\theta = \pi$, horizontal line
$$y = 0$$

69. $$r = 4$$
$$r^2 = 16$$
$$x^2 + y^2 = 16$$

70. $$r = 10$$
$$r^2 = 100$$
$$x^2 + y^2 = 100$$

71. $$r = -3\csc\theta$$
$$r\sin\theta = -3$$
$$y = -3$$

72. $$r = 2\sec\theta$$
$$r\cos\theta = 2$$
$$x = 2$$

73. $$r^2 = \cos\theta$$
$$r^3 = r\cos\theta$$
$$(x^2 + y^2)^{3/2} = x$$
$$x^2 + y^2 = x^{2/3}$$
$$(x^2 + y^2)^3 = x^2$$

74. $$r^2 = \sin 2\theta = 2\sin\theta\cos\theta$$
$$r^2 = 2\left(\frac{y}{r}\right)\left(\frac{x}{r}\right) = \frac{2xy}{r^2}$$
$$r^4 = 2xy$$
$$(x^2 + y^2)^2 = 2xy$$

75. $$r = 2\sin 3\theta$$
$$r = 2(3\sin\theta - 4\sin^3\theta)$$
$$r^4 = 6r^3\sin\theta - 8r^3\sin^3\theta$$
$$(x^2 + y^2)^2 = 6(x^2 + y^2)y - 8y^3$$
$$(x^2 + y^2)^2 = 6x^2y - 2y^3$$

76. $$r = 3\cos 2\theta$$
$$r = 3(\cos^2\theta - \sin^2\theta)$$
$$r^3 = 3(r^2\cos^2\theta - r^2\sin^2\theta)$$
$$(x^2 + y^2)^{3/2} = 3(x^2 - y^2) \text{ or } (x^2 + y^2)^3 = 9(x^2 - y^2)^2$$

77. $$r = \frac{1}{1 - \cos\theta}$$
$$r - r\cos\theta = 1$$
$$\sqrt{x^2 + y^2} - x = 1$$
$$x^2 + y^2 = 1 + 2x + x^2$$
$$y^2 = 2x + 1$$

78. $$r = \frac{2}{1 + \sin\theta}$$
$$r + r\sin\theta = 2$$
$$\sqrt{x^2 + y^2} + y = 2$$
$$x^2 + y^2 = (2 - y)^2$$
$$x^2 + y^2 = 4 - 4y + y^2$$
$$x^2 + 4y - 4 = 0$$

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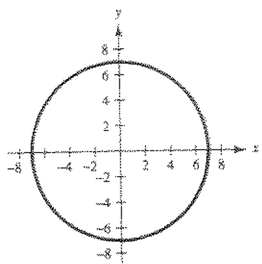
79.
$$r = \frac{6}{2 - 3\sin\theta}$$
$$r(2 - 3\sin\theta) = 6$$
$$2r = 6 + 3r\sin\theta$$
$$2\left(\pm\sqrt{x^2 + y^2}\right) = 6 + 3y$$
$$4(x^2 + y^2) = (6 + 3y)^2$$
$$4x^2 + 4y^2 = 36 + 36y + 9y^2$$
$$4x^2 - 5y^2 - 36y - 36 = 0$$

80.
$$r = \frac{6}{2\cos\theta - 3\sin\theta}$$
$$r = \frac{6}{2(x/r) - 3(y/r)}$$
$$r = \frac{6r}{2x - 3y}$$
$$1 = \frac{6}{2x - 3y}$$
$$2x - 3y = 6$$

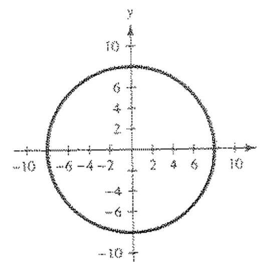
81.
$$r = 7$$
$$r^2 = 49$$
$$x^2 + y^2 = 49$$

The graph is a circle centered at the origin with radius 7.



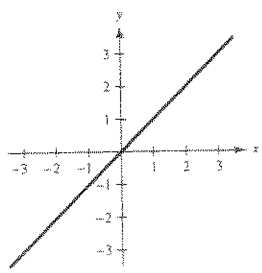
82.
$$r = 8$$
$$r^2 = 64$$
$$x^2 + y^2 = 64$$

Circle of radius 8 centered at origin



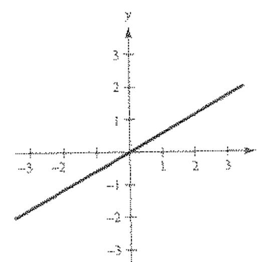
83.
$$\theta = \frac{\pi}{4}$$
$$\tan\theta = \tan\frac{\pi}{4} = 1 = \frac{y}{x}$$
$$y = x$$

The graph is the line $y = x$, which makes an angle of $\theta = \pi/4$ with the positive x-axis.



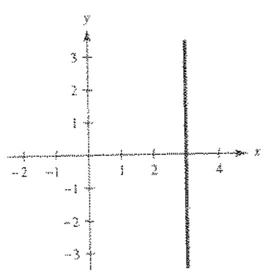
84.
$$\theta = \frac{7\pi}{6}$$
$$\frac{y}{x} = \tan\theta = \tan\frac{7\pi}{6} = \frac{\sqrt{3}}{3}$$
$$3y - \sqrt{3}x = 0$$

Line through origin making angle of $\pi/6$ with positive x-axis



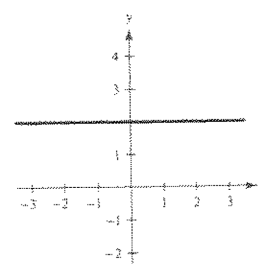
85.
$$r = 3\sec\theta$$
$$r\cos\theta = 3$$
$$x = 3$$
$$x - 3 = 0$$

Vertical line



86.
$$r = 2\csc\theta$$
$$r\sin\theta = 2$$
$$y = 2$$
$$y - 2 = 0$$

Horizontal line through $(0, 2)$



87. True, the distances from the origin are the same.

88. False. For instance when $r = 0$, any value of θ gives the same point.

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89. (a) $(r_1, \theta_1) = (x_1, y_1)$ where $x_1 = r_1 \cos \theta_1$ and $y_1 = r_1 \sin \theta_1$.

$(r_2, \theta_2) = (x_2, y_2)$ where $x_2 = r_2 \cos \theta_2$ and $y_2 = r_2 \sin \theta_2$.

Then $x_1^2 + y_1^2 = r_1^2 \cos^2 \theta_1 + r_1^2 \sin^2 \theta_1 = r_1^2$ and $x_2^2 + y_2^2 = r_2^2$. Thus,

$$\begin{aligned} d &= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \\ &= \sqrt{x_1^2 - 2x_1x_2 + x_2^2 + y_1^2 - 2y_1y_2 + y_2^2} \\ &= \sqrt{(x_1^2 + y_1^2) + (x_2^2 + y_2^2) - 2(x_1x_2 + y_1y_2)} \\ &= \sqrt{r_1^2 + r_2^2 - 2(r_1r_2 \cos \theta_1 \cos \theta_2 + r_1r_2 \sin \theta_1 \sin \theta_2)} \\ &= \sqrt{r_1^2 + r_2^2 - 2r_1r_2 \cos(\theta_1 - \theta_2)}. \end{aligned}$$

(b) If $\theta_1 = \theta_2$, the points are on the same line through the origin. In this case,

$d = \sqrt{r_1^2 + r_2^2 - 2r_1r_2 \cos(0)} = \sqrt{(r_1 - r_2)^2} = |r_1 - r_2|$.

(c) If $\theta_1 - \theta_2 = 90°$, $d = \sqrt{r_1^2 + r_2^2}$, the Pythagorean Theorem.

(d) For instance, $\left(3, \frac{\pi}{6}\right), \left(4, \frac{\pi}{3}\right)$ gives $d \approx 2.053$ and $\left(-3, \frac{7\pi}{6}\right), \left(-4, \frac{4\pi}{3}\right)$ gives $d \approx 2.053$. (Same!)

90. Answers will vary.

91. $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc} = \dfrac{19^2 + 25^2 - 13^2}{2(19)(25)} = 0.86$

$A \approx 30.7°$

$\cos B = \dfrac{a^2 + c^2 - b^2}{2ac} = \dfrac{13^2 + 25^2 - 19^2}{2(13)(25)} = 0.66615$

$B \approx 48.2°$

$C \approx 180° - 30.7° - 48.2° = 101.1°$

92. $A = 24°, a = 10, b = 6$

$\sin B = \dfrac{b \sin A}{a} \approx 0.2440 \implies B \approx 14.1°$

$C = 180° - A - B \approx 141.9°$

$c = \dfrac{a \sin C}{\sin A} \approx 15.17$

93. $B = 180° - 56° - 38° = 86°$

$\dfrac{a}{\sin A} = \dfrac{c}{\sin C} \implies a = \dfrac{c \sin A}{\sin C} = \dfrac{12 \sin(56°)}{\sin(38°)} \approx 16.16$

$\dfrac{b}{\sin B} = \dfrac{c}{\sin C} \implies b = \dfrac{c \sin B}{\sin C} = \dfrac{12 \sin(86°)}{\sin(38°)} \approx 19.44$

94. $B = 71°, a = 21, c = 29$

$b^2 = a^2 + c^2 - 2ac \cos B \approx 885.458 \implies b \approx 29.76$

$\sin C = c\dfrac{\sin B}{b} \approx 0.9214 \implies C \approx 67.1°$

$A = 180° - B - C \approx 41.9°$

95. $c^2 = a^2 + b^2 - 2ab \cos C$

$= 8^2 + 4^2 - 2(8)(4) \cos(35°)$

≈ 27.57

$c \approx 5.25$

$\dfrac{b}{\sin B} = \dfrac{c}{\sin C} \implies \sin B = \dfrac{b \sin C}{c} = \dfrac{4 \sin(35°)}{5.25}$

$\implies B \approx 25.9°$

$A = 180° - B - C = 119.1°$

96. $B = 64°, b = 52, c = 44$

$\sin C = \dfrac{c \sin B}{b} \approx 0.7605 \implies C \approx 49.5°$

$A = 180° - B - C \approx 66.5°$

$a = \dfrac{b \sin A}{\sin B} \approx 53.06$

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97. By Cramer's Rule,

$$D = \begin{vmatrix} 5 & -7 \\ -3 & 1 \end{vmatrix} = 5 - 21 = -16$$

$$D_x = \begin{vmatrix} -11 & -7 \\ -3 & 1 \end{vmatrix} = -11 - 21 = -32$$

$$D_y = \begin{vmatrix} 5 & -11 \\ -3 & -3 \end{vmatrix} = -15 - 33 = -48$$

$$x = \frac{D_x}{D} = \frac{-32}{-16} = 2,\ y = \frac{D_y}{D} = \frac{-48}{-16} = 3$$

Solution: $(2, 3)$

98. By Cramer's Rule,

$$D = \begin{vmatrix} 3 & 5 \\ 4 & -2 \end{vmatrix} = -6 - 20 = -26$$

$$D_x = \begin{vmatrix} -10 & 5 \\ -5 & -2 \end{vmatrix} = -20 - (-25) = 5$$

$$D_y = \begin{vmatrix} 3 & 10 \\ 4 & -5 \end{vmatrix} = -15 - 40 = -55$$

$$x = \frac{D_x}{D} = -\frac{5}{26},\ y = \frac{D_y}{D} = \frac{-55}{-26} = \frac{55}{26}$$

Solution: $\left(-\frac{5}{26}, \frac{55}{26}\right)$

99. By Cramer's Rule,

$$D = \begin{vmatrix} 3 & -2 & 1 \\ 2 & 1 & -3 \\ 1 & -3 & 9 \end{vmatrix} = 35$$

$$D_a = \begin{vmatrix} 0 & -2 & 1 \\ 0 & 1 & -3 \\ 0 & -3 & 9 \end{vmatrix} = 0$$

$$D_b = \begin{vmatrix} 3 & 0 & 1 \\ 2 & 0 & -3 \\ 1 & 0 & 9 \end{vmatrix} = 0$$

$$D_c = \begin{vmatrix} 3 & -2 & 0 \\ 2 & 1 & 0 \\ 1 & -3 & 0 \end{vmatrix} = 0$$

$$a = \frac{D_a}{D} = 0,\ b = \frac{D_b}{D} = 0,\ c = \frac{D_c}{D} = 0$$

Solution: $(0, 0, 0)$

100. By Cramer's Rule,

$$D = \begin{vmatrix} 5 & 7 & 9 \\ 1 & -2 & -3 \\ 8 & -2 & 1 \end{vmatrix} = -89$$

$$D_u = \begin{vmatrix} 15 & 7 & 9 \\ 7 & -2 & -3 \\ 0 & -2 & 1 \end{vmatrix} = -295$$

$$D_v = \begin{vmatrix} 5 & 15 & 9 \\ 1 & 7 & -3 \\ 8 & 0 & 1 \end{vmatrix} = -844$$

$$D_w = \begin{vmatrix} 5 & 7 & 15 \\ 1 & -2 & 7 \\ 8 & -2 & 0 \end{vmatrix} = 672$$

$$u = \frac{D_u}{D} = \frac{-295}{-89} = \frac{295}{89},\ v = \frac{D_v}{D} = \frac{-844}{-89} = \frac{844}{89},$$

$$w = \frac{D_w}{D} = \frac{672}{-89} = \frac{-672}{89}$$

Solution: $\left(\frac{295}{89}, \frac{844}{89}, -\frac{672}{89}\right)$

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101. By Cramer's Rule,

$$D = \begin{vmatrix} -1 & 1 & 2 \\ 2 & 3 & 1 \\ 5 & 4 & 2 \end{vmatrix} = -15$$

$$D_x = \begin{vmatrix} 1 & 1 & 2 \\ -2 & 3 & 1 \\ 4 & 4 & 2 \end{vmatrix} = -30$$

$$D_y = \begin{vmatrix} -1 & 1 & 2 \\ 2 & -2 & 1 \\ 5 & 4 & 2 \end{vmatrix} = 45$$

$$D_z = \begin{vmatrix} -1 & 1 & 1 \\ 2 & 3 & -2 \\ 5 & 4 & 4 \end{vmatrix} = -45$$

$$x = \frac{D_x}{D} = \frac{-30}{-15} = 2,\ y = \frac{D_y}{D} = \frac{45}{-15} = -3,$$

$$z = \frac{D_z}{D} = \frac{-45}{-15} = 3$$

Solution: $(2, -3, 3)$

102. Cramer's Rule does not apply because

$$D = \begin{vmatrix} 2 & 1 & 2 \\ 2 & 2 & 0 \\ 2 & -1 & 6 \end{vmatrix} = 0.$$

Use elimination to solve the system.

$$\left[\begin{array}{ccccc} 2 & 1 & 2 & \vdots & 4 \\ 2 & 2 & 0 & \vdots & 5 \\ 2 & -1 & 6 & \vdots & 2 \end{array}\right] \rightarrow$$

$$\begin{array}{r} \\ -R_1 + R_2 \\ -R_1 + R_3 \end{array}\left[\begin{array}{ccccc} 2 & 1 & 2 & \vdots & 4 \\ 0 & 1 & -2 & \vdots & 1 \\ 0 & -2 & 4 & \vdots & -2 \end{array}\right] \rightarrow$$

$$\begin{array}{r} -R_2 + R_1 \\ \\ 2R_2 + R_3 \end{array}\left[\begin{array}{ccccc} 2 & 0 & 4 & \vdots & 3 \\ 0 & 1 & -2 & \vdots & 1 \\ 0 & 0 & 0 & \vdots & 0 \end{array}\right]$$

Let $x_3 = a$, then $x_2 = 2a + 1$ and $x_1 = -2a + \frac{3}{2}$.

Solution: $\left(-2a + \frac{3}{2}, 2a + 1, a\right)$

103. Points: $(4, -3), (6, -7), (-2, -1)$

$$\begin{vmatrix} 4 & -3 & 1 \\ 6 & -7 & 1 \\ -2 & -1 & 1 \end{vmatrix} = -20 \neq 0$$

The points are not collinear.

104. Points: $(-2, 4), (0, 1), (4, -5)$

$$\begin{vmatrix} -2 & 4 & 1 \\ 0 & 1 & 1 \\ 4 & -5 & 1 \end{vmatrix} = 0 \implies \text{collinear}$$

105. Points: $(-6, -4), (-1, -3), (1.5, -2.5)$

$$\begin{vmatrix} -6 & -4 & 1 \\ -1 & -3 & 1 \\ 1.5 & -2.5 & 1 \end{vmatrix} = 0$$

The points are collinear.

106. Points: $(-2.3, 5), (-0.5, 0), (1.5, -3)$

$$\begin{vmatrix} -2.3 & 5 & 1 \\ -0.5 & 0 & 1 \\ 1.5 & -3 & 1 \end{vmatrix} = 4.6 \implies \text{not collinear}$$

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Section 9.7 Graphs of Polar Equations

- When graphing polar equations:
  1. Test for symmetry
     (a) $\theta = \pi/2$: Replace (r, θ) by $(r, \pi - \theta)$ or $(-r, -\theta)$.
     (b) Polar axis: Replace (r, θ) by $(r, -\theta)$ or $(-r, \pi - \theta)$.
     (c) Pole: Replace (r, θ) by $(r, \pi + \theta)$ or $(-r, \theta)$.
     (d) $r = f(\sin \theta)$ is symmetric with respect to the line $\theta = \pi/2$.
     (e) $r = f(\cos \theta)$ is symmetric with respect to the polar axis.
  2. Find the θ values for which $|r|$ is maximum.
  3. Find the θ values for which $r = 0$.
  4. Know the different types of polar graphs.

(a) Limaçons	(b) Rose curves, $n \geq 2$	(c) Circles	(d) Lemniscates
$r = a \pm b \cos \theta$	$r = a \cos n\theta$	$r = a \cos \theta$	$r^2 = a^2 \cos 2\theta$
$r = a \pm b \sin \theta$	$r = a \sin n\theta$	$r = a \sin \theta$	$r^2 = a^2 \sin 2\theta$
		$r = a$	

- You should be able to graph polar equations of the form $r = f(\theta)$ with your graphing utility. If your utility does not have a polar mode, use

  $x = f(t) \cos t$

  $y = f(t) \sin t$

  in parametric mode.

Vocabulary Check

1. $\theta = \dfrac{\pi}{2}$
2. polar axis
3. convex limaçon
4. circle
5. lemniscate
6. cardioid

1. $r = 3 \cos 2\theta$ is a rose curve.
2. Cardioid
3. Lemniscate
4. $r = 3 \cos \theta$ is a circle.
5. $r = 6 \sin 2\theta$ is a rose curve.
6. Limaçon
7. The graph is symmetric about the line $\theta = \pi/2$, and passes through $(r, \theta) = (3, 3\pi/2)$. Matches (a).
8. The graph is symmetric about the polar axis and passes through $(r, \theta) = (3, 0)$. Matches (c).
9. The graph has four leaves. Matches (c).
10. The graph has three leaves. Matches (d).

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11. $r = 14 + 4\cos\theta$

$\theta = \frac{\pi}{2}$: $-r = 14 + 4\cos(-\theta)$

$-r = 14 + 4\cos\theta$

Not an equivalent equation

$r = 14 + 4\cos(\pi - \theta)$

$r = 14 + 4(\cos\pi\cos\theta + \sin\pi\sin\theta)$

$r = 14 - 4\cos\theta$

Not an equivalent equation

Polar axis: $r = 14 + 4\cos(-\theta)$

$r = 14 + 4\cos\theta$

Equivalent equation

Pole: $-r = 14 + 4\cos\theta$

Not an equivalent equation

$r = 14 + 4\cos(\pi + \theta)$

$r = 14 - 4\cos\theta$

Not an equivalent equation

Answer: Symmetric with respect to polar axis

12. $r = 12\cos 3\theta$

$\theta = \frac{\pi}{2}$: $-r = 12\cos(3(-\theta))$

$-r = 12\cos 3\theta$

Not an equivalent equation

$r = 12\cos(3(\pi - \theta))$

$r = -12\cos 3\theta$

Not an equivalent equation

Polar axis: $r = 12\cos(3(-\theta))$

$r = 12\cos 3\theta$

Equivalent equation

Pole: $-r = 12\cos 3\theta$

Not an equivalent equation

$r = 12\cos(3(\pi + \theta))$

$r = -12\cos 3\theta$

Not an equivalent equation

Answer: Symmetric with respect to polar axis

13. $r = \frac{4}{1 + \sin\theta}$

$\theta = \frac{\pi}{2}$: $r = \frac{4}{1 + \sin(\pi - \theta)}$

$r = \frac{4}{1 + \sin\pi\cos\theta - \cos\pi\sin\theta}$

$r = \frac{4}{1 + \sin\theta}$

Equivalent equation

Polar axis: $r = \frac{4}{1 + \sin(-\theta)}$

$r = \frac{4}{1 - \sin\theta}$

Not an equivalent equation

$-r = \frac{4}{1 + \sin(\pi - \theta)}$

$-r = \frac{4}{1 + \sin\theta}$

Not an equivalent equation

Pole: $-r = \frac{4}{1 + \sin\theta}$

Not an equivalent equation

$r = \frac{4}{1 + \sin(\pi + \theta)}$

$r = \frac{4}{1 - \sin\theta}$

Not an equivalent equation

Answer: Symmetric with respect to $\theta = \frac{\pi}{2}$

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14. $r = \dfrac{2}{1 - \cos\theta}$

$\theta = \dfrac{\pi}{2}$: $-r = \dfrac{2}{1 - \cos(-\theta)}$

$-r = \dfrac{2}{1 - \cos\theta}$

Not an equivalent equation

$r = \dfrac{2}{1 - \cos(\pi - \theta)}$

$r = \dfrac{2}{1 - (\cos\pi\cos\theta + \sin\pi\sin\theta)}$

$r = \dfrac{2}{1 + \cos\theta}$

Not an equivalent equation

Polar axis: $r = \dfrac{2}{1 - \cos(-\theta)}$

$r = \dfrac{2}{1 - \cos\theta}$

Equivalent equation

Pole: $-r = \dfrac{2}{1 - \cos\theta}$

Not an equivalent equation

$r = \dfrac{2}{1 - \cos(\pi + \theta)}$

$r = \dfrac{2}{1 - (\cos\pi\cos\theta - \sin\pi\sin\theta)}$

$r = \dfrac{2}{1 + \cos\theta}$

Not an equivalent equation

Answer: Symmetric with respect to the polar axis

15. $r = 6\sin\theta$

$\theta = \dfrac{\pi}{2}$: $-r = 6\sin(-\theta)$

$r = 6\sin\theta$

Equivalent equation

Polar axis: $r = 6\sin(-\theta)$

$r = -6\sin\theta$

Not an equivalent equation

$-r = 6\sin(\pi - \theta)$

$-r = 6(\sin\pi\cos\theta - \cos\pi\sin\theta)$

$-r = 6\sin\theta$

Not an equivalent equation

Pole: $-r = 6\sin\theta$

Not an equivalent equation

$r = 6\sin(\pi + \theta)$

$r = -6\sin\theta$

Not an equivalent equation

Answer: Symmetric with respect to $\theta = \dfrac{\pi}{2}$

16. $r = 4\csc\theta\cos\theta = 4\cot\theta$

$\theta = \dfrac{\pi}{2}$: $-r = 4\cot(-\theta)$

$r = 4\cot\theta$

Equivalent equation

Polar axis: $-r = 4\cot(\pi - \theta)$

$-r = 4\cot(-\theta)$

$r = 4\cot\theta$

Equivalent equation

Pole: $r = 4\cot(\pi + \theta)$

$r = 4\cot\theta$

Equivalent equation

Answer: Symmetric with respect to $\theta = \dfrac{\pi}{2}$, polar axis and pole

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17. $r^2 = 16 \sin 2\theta$

$\theta = \dfrac{\pi}{2}$: $(-r)^2 = 16 \sin(2(-\theta))$

$r^2 = -16 \sin 2\theta$

Not an equivalent equation

$r^2 = 16 \sin(2(\pi - \theta))$

$r^2 = 16 \sin(2\pi - 2\theta)$

$r^2 = -16 \sin 2\theta$

Not an equivalent equation

Polar axis: $r^2 = 16 \sin(2(-\theta))$

$r^2 = -16 \sin 2\theta$

Not an equivalent equation

$(-r)^2 = 16 \sin(2(\pi - \theta))$

$r^2 = -16 \sin 2\theta$

Not an equivalent equation

Pole: $(-r)^2 = 16 \sin(2\theta)$

$r^2 = 16 \sin 2\theta$

Equivalent equation

Answer: Symmetric with respect to pole

18. $r^2 = 25 \cos 4\theta$

$\theta = \dfrac{\pi}{2}$: $(-r)^2 = 25 \cos(4(-\theta))$

$r^2 = 25 \cos 4\theta$

Equivalent equation

Polar axis: $r^2 = 25 \cos(4(-\theta))$

$r^2 = 25 \cos 4\theta$

Equivalent equation

Pole: $(-r)^2 = 25 \cos 4\theta$

$r^2 = 25 \cos 4\theta$

Equivalent equation

Answer: Symmetric with respect to $\theta = \dfrac{\pi}{2}$, polar axis and pole

19. $|r| = |10(1 - \sin \theta)|$

$= 10|1 - \sin \theta| \le 10(2) = 20$

$|1 - \sin \theta| = 2$

$1 - \sin \theta = 2$ or $1 - \sin \theta = -2$

$\sin \theta = -1$ $\qquad \sin \theta = 3$

$\theta = \dfrac{3\pi}{2}$ $\qquad$ Not possible

Maximum: $|r| = 20$ when $\theta = \dfrac{3\pi}{2}$

$r = 0$ when $1 - \sin \theta = 0$

$\sin \theta = 1$

$\theta = \dfrac{\pi}{2}$

20. $|r| = |6 + 12 \cos \theta| \le |6| + |12 \cos \theta|$

$= 6 + 12|\cos \theta| \le 18$

$\cos \theta = 1$

$\theta = 0$

Maximum: $|r| = 18$ when $\theta = 0$

Zero: $r = 0$ when $\theta = \dfrac{2\pi}{3}, \dfrac{4\pi}{3}$

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21. $|r| = |4 \cos 3\theta| = 4\,|\cos 3\theta| \le 4$

$|\cos 3\theta| = 1$

$\cos 3\theta = \pm 1$

$\theta = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi$

Maximum: $|r| = 4$ when $\theta = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi$

$r = 0$ when $\cos 3\theta = 0$

$\theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$

22. $r = \sin 2\theta$

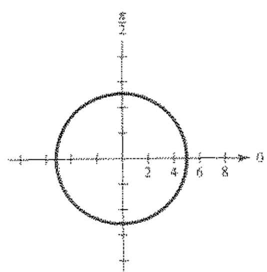
$|r| = |\sin 2\theta|$

Maximum: $|r| = 1$ when $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

Zero: $r = 0$ when $\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$

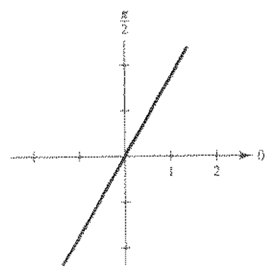
23. $r = 5$

Circle



24. $\theta = -\frac{5\pi}{3}$

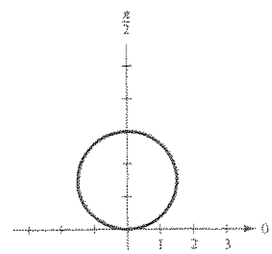
Line



25. $r = 3 \sin \theta$

Symmetric with respect to $\theta = \pi/2$

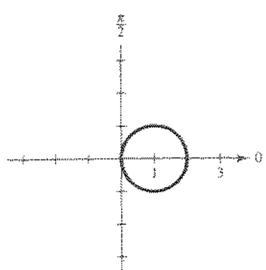
Circle with radius of $3/2$



26. $r = 2 \cos \theta$

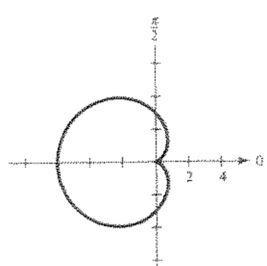
Circle

Radius: 1, center: $(1, 0)$



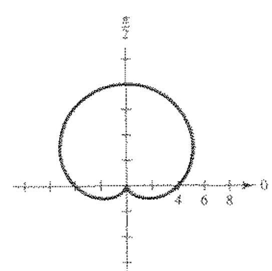
27. $r = 3(1 - \cos \theta)$

Cardioid



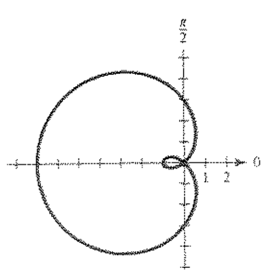
28. $r = 4(1 + \sin \theta)$

Cardioid



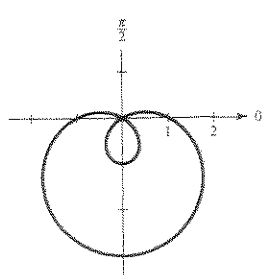
29. $r = 3 - 4 \cos \theta$

Limaçon



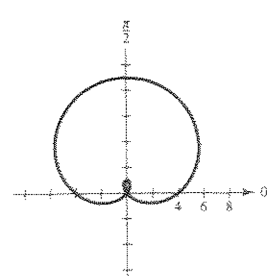
30. $r = 1 - 2 \sin \theta$

Limaçon with inner loop



31. $r = 4 + 5 \sin \theta$

Limaçon



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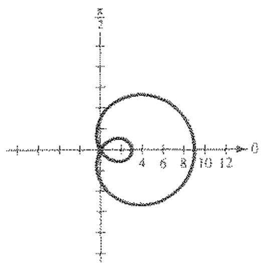
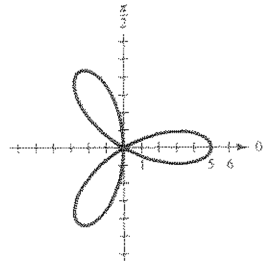
32. $r = 3 + 6 \cos \theta$

Limaçon



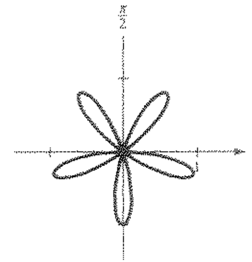
33. $r = 5 \cos 3\theta$

Rose curve



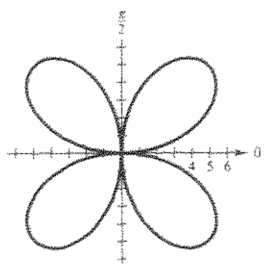
34. $r = -\sin 5\theta$

Rose curve



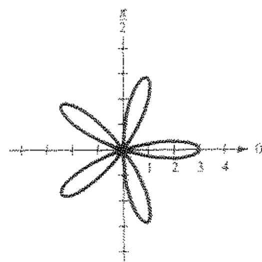
35. $r = 7 \sin 2\theta$

Rose curve, four petals

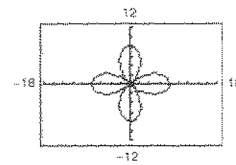


36. $r = 3 \cos 5\theta$

Rose curve, five petals

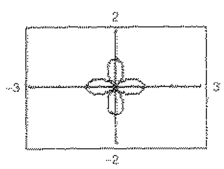


37. $r = 8 \cos 2\theta$



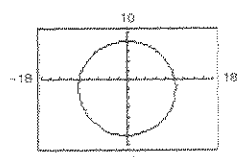
$0 \le \theta < 2\pi$

38.



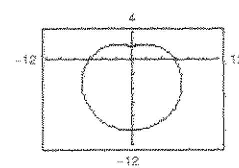
$0 \le \theta \le 2\pi$

39. $r = 2(5 - \sin \theta)$



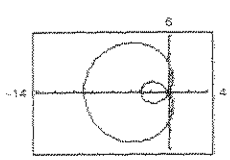
$0 \le \theta < 2\pi$

40.

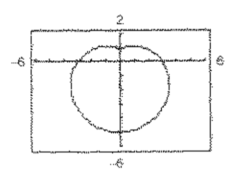


$0 \le \theta \le 2\pi$

41. $r = 3 - 6 \cos \theta,\ 0 \le \theta \le 2\pi$

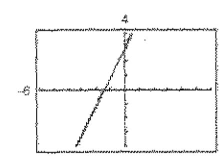


42.

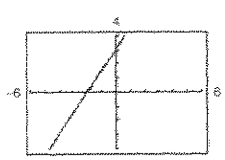


$0 \le \theta \le 2\pi$

43. $r = \dfrac{3}{\sin \theta - 2 \cos \theta},\ 0 \le \theta \le \dfrac{\pi}{2}$

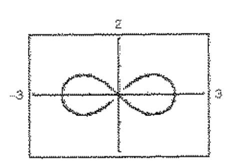


44.



$0 \le \theta \le 2\pi$

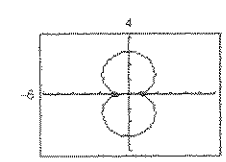
45. $r^2 = 4 \cos 2\theta,\ -2\pi \le \theta \le 2\pi$



46. $r^2 = 9 \sin \theta$

$r = \pm 3\sqrt{\sin \theta}$

Graph both functions using $0 \le \theta \le 2\pi$.



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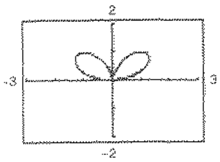
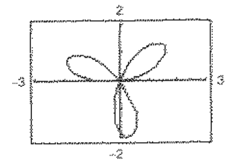
47. $r = 4 \sin \theta \cos^2 \theta,\ 0 \le \theta \le \pi$

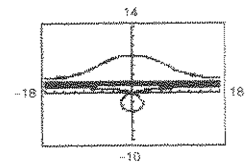


48.



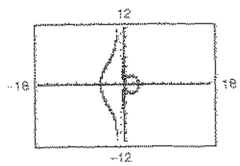
$0 \le \theta \le \pi$

49. $r = 2 \csc \theta + 6$



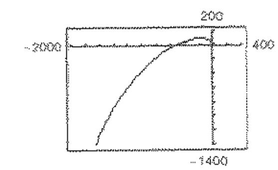
$0 \le \theta < 2\pi$

50.



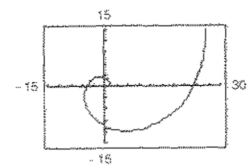
$0 \le \theta \le 2\pi$

51. $r = e^{2\theta}$



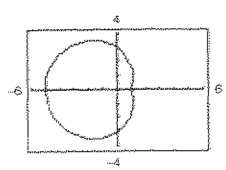
Answers will vary.

52. $r = e^{\theta/2}$

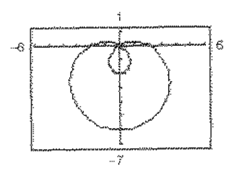


Answers will vary.

53. $r = 3 - 2\cos\theta,\ 0 \le \theta < 2\pi$

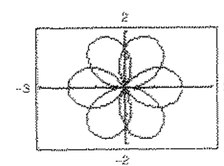


54.

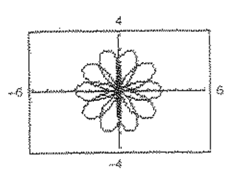


$0 \le \theta < 2\pi$

55. $r = 2\cos\left(\frac{3\theta}{2}\right),\ 0 \le \theta < 4\pi$



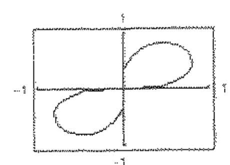
56.



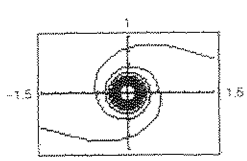
$0 \le \theta < 4\pi$

57. $r^2 = \sin 2\theta,\ 0 \le \theta < \frac{\pi}{2}$

$\left(\text{Use } r_1 = \sqrt{\sin 2\theta} \text{ and } r_2 = -\sqrt{\sin 2\theta}.\right)$



58.

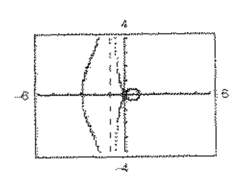


$0 < \theta < \infty$

$r = \dfrac{\pm 1}{\sqrt{\theta}}$

59. $r = 2 - \sec\theta$

$x = -1$ is an asymptote.



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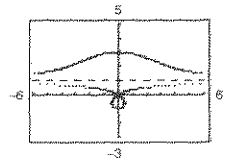
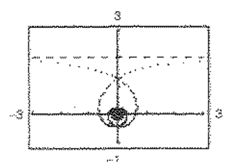
60.
$$\begin{aligned}
r &= 2 + \csc\theta = 2 + \frac{1}{\sin\theta}\\
r\sin\theta &= 2\sin\theta + 1\\
r(r\sin\theta) &= 2r\sin\theta + r\\
\left(\pm\sqrt{x^2+y^2}\right)(y) &= 2y + \left(\pm\sqrt{x^2+y^2}\right)\\
\left(\pm\sqrt{x^2+y^2}\right)(y-1) &= 2y\\
\left(\pm\sqrt{x^2+y^2}\right) &= \frac{2y}{y-1}\\
x^2+y^2 &= \frac{4y^2}{(y-1)^2}\\
x^2 &= \frac{y^2(3+2y-y^2)}{(y-1)^2}\\
x &= \pm\sqrt{\frac{y^2(3+2y-y^2)}{(y-1)^2}} = \pm\left|\frac{y}{y-1}\right|\sqrt{3+2y-y^2}
\end{aligned}$$



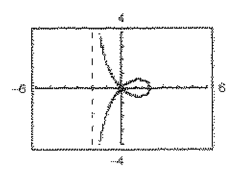
The graph has an asymptote at $y = 1$.

61. $r = \dfrac{2}{\theta}$

$y = 2$ is an asymptote.



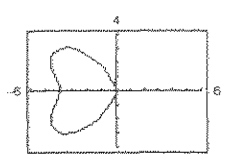
62.



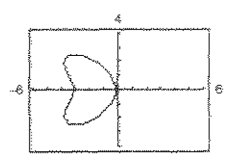
63. True. It has five petals.

64. False. For example, let $r = \cos 3\theta$.

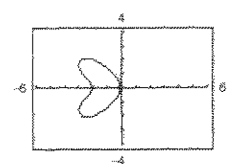
65. $r = \cos(5\theta) + n\cos\theta,\ 0 \le \theta < \pi$; Answers will vary.



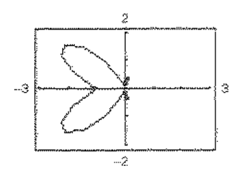
$n = -5$



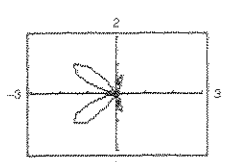
$n = -4$



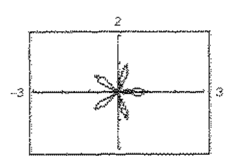
$n = -3$



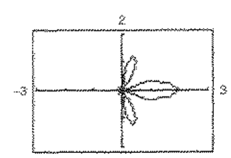
$n = -2$



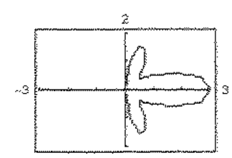
$n = -1$



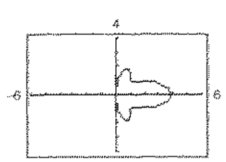
$n = 0$



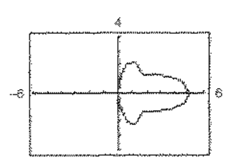
$n = 1$



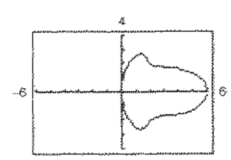
$n = 2$



$n = 3$



$n = 4$



$n = 5$

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66. The graph of $r = f(\theta)$ is rotated about the pole through an angle ϕ. Let (r, θ) be any point on the graph of $r = f(\theta)$. Then $(r, \theta + \phi)$ is rotated through the angle ϕ, and since $r = f((\theta + \phi) - \phi) = f(\theta)$, it follows that $(r, \theta + \phi)$ is on the graph of $r = f(\theta - \phi)$.

67. Use the result of Exercise 66.

(a) Rotation: $\phi = \dfrac{\pi}{2}$

Original graph: $r = f(\sin\theta)$

Rotated graph: $r = f\left(\sin\left(\theta - \dfrac{\pi}{2}\right)\right) = f(-\cos\theta)$

(b) Rotation: $\phi = \pi$

Original graph: $r = f(\sin\theta)$

Rotated graph: $r = f(\sin(\theta - \pi)) = f(-\sin\theta)$

(c) Rotation: $\phi = \dfrac{3\pi}{2}$

Original graph: $r = f(\sin\theta)$

Rotated graph: $r = f\left(\sin\left(\theta - \dfrac{3\pi}{2}\right)\right) = f(\cos\theta)$

68. (a) $r = 2 - \sin\left(\theta - \dfrac{\pi}{4}\right)$

$= 2 - \dfrac{\sqrt{2}}{2}(\sin\theta - \cos\theta)$

(b) $r = 2 - \sin\left(\theta - \dfrac{\pi}{2}\right)$

$= 2 + \cos\theta$

(c) $r = 2 - \sin(\theta - \pi)$

$= 2 + \sin\theta$

(d) $r = 2 - \sin\left(\theta - \dfrac{3\pi}{2}\right)$

$= 2 - \cos\theta$

69. (a) $r = 2\sin\left[2\left(\theta - \dfrac{\pi}{6}\right)\right]$

$= 2\sin\left(2\theta - \dfrac{\pi}{3}\right)$

$= \sin 2\theta - \sqrt{3}\cos 2\theta$

(b) $r = 2\sin\left[2\left(\theta - \dfrac{\pi}{2}\right)\right]$

$= 2\sin(2\theta - \pi)$

$= -2\sin 2\theta$

$= -4\sin\theta\cos\theta$

(c) $r = 2\sin\left[2\left(\theta - \dfrac{2\pi}{3}\right)\right]$

$= 2\sin\left(2\theta - \dfrac{4\pi}{3}\right)$

$= \sqrt{3}\cos 2\theta - \sin 2\theta$

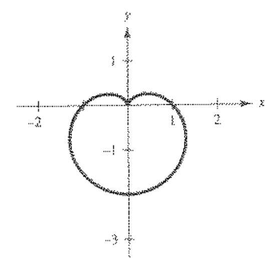
(d) $r = 2\sin[2(\theta - \pi)]$

$= 2\sin(2\theta - 2\pi)$

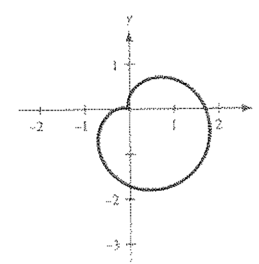
$= 2\sin 2\theta$

$= 4\sin\theta\cos\theta$

70. (a) $r = 1 - \sin\theta$



(b) $r = 1 - \sin\left(\theta - \dfrac{\pi}{4}\right)$



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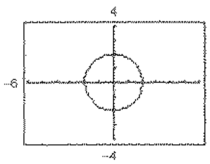
71. $r = 2 + k\cos\theta$

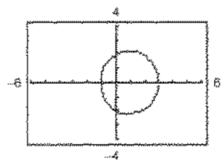
$k = 0$

Circle



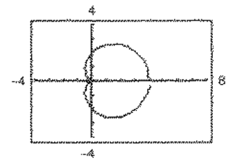
$k = 1$

Convex limaçon



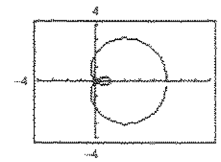
$k = 2$

Cardioid



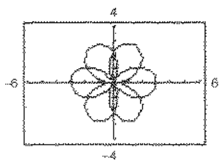
$k = 3$

Limaçon with inner loop



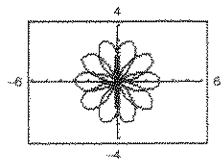
72. $r = 3\sin k\theta$

(a)



$k = 1.5$: $0 \le \theta < 4\pi$

(b)



$k = 2.5$: $0 \le \theta < 4\pi$

(c) Yes. Answers will vary.

Section 9.8 Polar Equations of Conics

- The graph of a polar equation of the form

  $$r = \frac{ep}{1 \pm e\cos\theta} \quad \text{or} \quad r = \frac{ep}{1 \pm e\sin\theta}$$

  is a conic, where $e > 0$ is the eccentricity and $|p|$ is the distance between the focus (pole) and the directrix.

  (a) If $e < 1$, the graph is an ellipse.

  (b) If $e = 1$, the graph is a parabola.

  (c) If $e > 1$, the graph is a hyperbola.

- Guidelines for finding polar equations of conics:

  (a) Horizontal directrix above the pole: $r = \dfrac{ep}{1 + e\sin\theta}$

  (b) Horizontal directrix below the pole: $r = \dfrac{ep}{1 - e\sin\theta}$

  (c) Vertical directrix to the right of the pole: $r = \dfrac{ep}{1 + e\cos\theta}$

  (d) Vertical directrix to the left of the pole: $r = \dfrac{ep}{1 - e\cos\theta}$

Vocabulary Check

1. conic

2. eccentricity, e

3. (a) i (b) iii (c) ii

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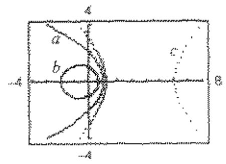
1. $r = \dfrac{2e}{1 + e\cos\theta}$

(a) Parabola

(b) Ellipse

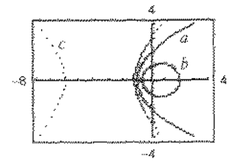
(c) Hyperbola



2. (a) Parabola

(b) Ellipse

(c) Hyperbola

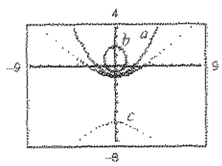


3. $r = \dfrac{2e}{1 - e\sin\theta}$

(a) Parabola

(b) Ellipse

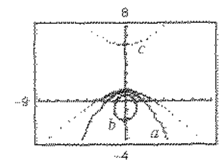
(c) Hyperbola



4. (a) Parabola

(b) Ellipse

(c) Hyperbola



5. $r = \dfrac{4}{1 - \cos\theta}$

$e = 1 \Longrightarrow$ parabola

Vertical directrix to left of pole

Matches (b).

6. $r = \dfrac{3}{2 - \cos\theta} = \dfrac{3/2}{1 - (1/2)\cos\theta}$

$e = \dfrac{1}{2} \Longrightarrow$ ellipse

Vertical directrix to left of pole

Matches (c).

7. $r = \dfrac{3}{2 + \cos\theta} = \dfrac{3/2}{1 + (1/2)\cos\theta}$

$e = \dfrac{1}{2} \Longrightarrow$ ellipse

Vertical directrix to right of pole

Matches (f).

8. $r = \dfrac{4}{1 - 3\sin\theta}$

$e = 3 \Longrightarrow$ hyperbola

Horizontal directrix below the pole.

Matches (e).

9. $r = \dfrac{3}{1 + 2\sin\theta}$

$e = 2 \Longrightarrow$ hyperbola

Horizontal directrix above the pole.

Matches (d).

10. $r = \dfrac{4}{1 + \sin\theta}$

$e = 1 \Longrightarrow$ parabola

Vertex: $\left(2, \dfrac{\pi}{2}\right)$

Matches (a).

11. $r = \dfrac{2}{1 - \cos\theta}$

$e = 1 \Longrightarrow$ parabola

Vertex: $(r, \theta) = (1, \pi)$

12. $r = \dfrac{2}{1 + \sin\theta}$

$e = 1 \Longrightarrow$ parabola

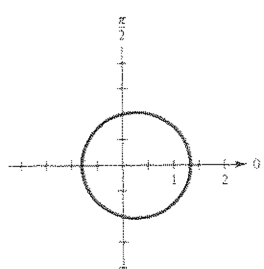
Vertex: $(1, \pi/2)$

13. $r = \dfrac{4}{4 - \cos\theta} = \dfrac{1}{1 - (1/4)\cos\theta}$

$e = \dfrac{1}{4}, p = 4,$ ellipse

Vertices:

$(r, \theta) = \left(\dfrac{4}{3}, 0\right), \left(\dfrac{4}{5}, \pi\right)$



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14. $r = \dfrac{7}{7 + \sin\theta} = \dfrac{1}{1 + (1/7)\sin\theta}$

$e = \dfrac{1}{7} \Longrightarrow$ ellipse

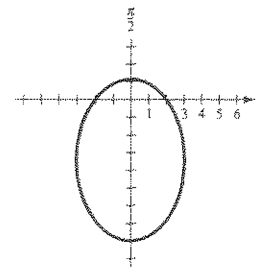
Vertices: $(r, \theta) = \left(\dfrac{7}{8}, \dfrac{\pi}{2}\right), \left(\dfrac{7}{6}, \dfrac{3\pi}{2}\right)$

15. $r = \dfrac{8}{4 + 3\sin\theta} = \dfrac{2}{1 + (3/4)\sin\theta}$

$e = \dfrac{3}{4} \Longrightarrow$ ellipse

Vertices:

$(r, \theta) = \left(\dfrac{8}{7}, \dfrac{\pi}{2}\right), \left(8, \dfrac{3\pi}{2}\right)$



16. $r = \dfrac{6}{3 - 2\cos\theta} = \dfrac{2}{1 - (2/3)\cos\theta}$

$e = \dfrac{2}{3} \Longrightarrow$ ellipse

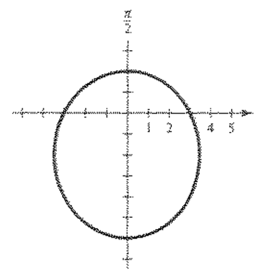
Vertices: $(6, 0), \left(\dfrac{6}{5}, \pi\right)$

17. $r = \dfrac{6}{2 + \sin\theta} = \dfrac{(1/2)(6)}{1 + (1/2)\sin\theta}$

$e = \dfrac{1}{2} \Longrightarrow$ ellipse

Vertices:

$\left(2, \dfrac{\pi}{2}\right), \left(6, \dfrac{3\pi}{2}\right)$



18. $r = \dfrac{5}{-1 + 2\cos\theta} = \dfrac{-5}{1 - 2\cos\theta}$

$e = 2 \Longrightarrow$ hyperbola

Vertices: $(5, 0), \left(-\dfrac{5}{3}, \pi\right)$

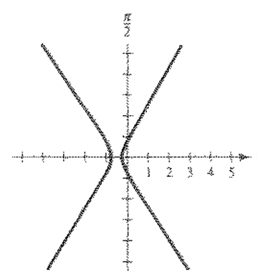
19. $r = \dfrac{3}{4 - 8\cos\theta} = \dfrac{3/4}{1 - 2\cos\theta}$

$e = 2 \Longrightarrow$ hyperbola

Hyperbola

Vertices:

$(r, \theta) = \left(-\dfrac{3}{4}, 0\right), \left(\dfrac{1}{4}, \pi\right)$



20. $r = \dfrac{10}{3 + 9\sin\theta} = \dfrac{10/3}{1 + 3\sin\theta}$

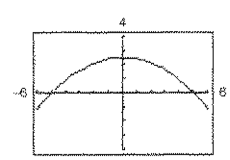
$e = 3 \Longrightarrow$ hyperbola

Vertices:

$(r, \theta) = \left(\dfrac{5}{6}, \dfrac{\pi}{2}\right), \left(-\dfrac{5}{3}, \dfrac{3\pi}{2}\right)$

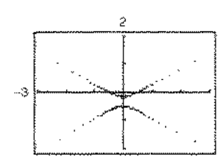
21. $r = \dfrac{-5}{1 - \sin\theta}$

Parabola



22. $r = \dfrac{-1}{2 + 4\sin\theta}$

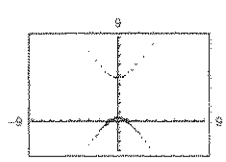
Hyperbola



23. $r = \dfrac{14}{14 + 17\sin\theta}$

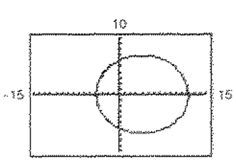
$= \dfrac{1}{1 + (17/14)\sin\theta}$

Hyperbola



24. $r = \dfrac{12}{2 - \cos\theta}$

Ellipse



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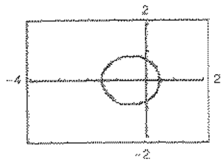
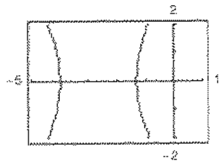
25.



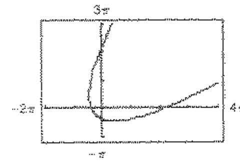
Ellipse

26.



Hyperbola

27.



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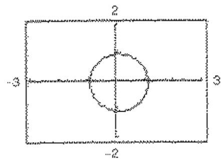

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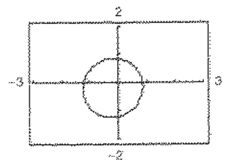

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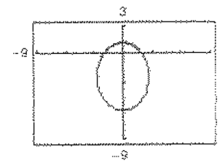

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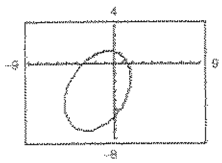
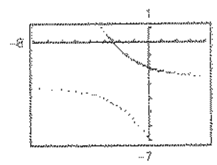


32.



33. $e = 1, x = -1, p = 1$

Vertical directrix to the left of the pole

$$r = \frac{1(1)}{1 - 1\cos\theta} = \frac{1}{1 - \cos\theta}$$

34. $e = 1, y = -4, p = 4$

Horizontal directrix below the pole

$$r = \frac{1(4)}{1 - (1)\sin\theta} = \frac{4}{1 - \sin\theta}$$

35. $e = \frac{1}{2}, y = 1, p = 1$

Horizontal directrix above the pole

$$r = \frac{(1/2)(1)}{1 + (1/2)\sin\theta} = \frac{1}{2 + \sin\theta}$$

36. $e = \frac{3}{4}, y = -4, p = 4$

Horizontal directrix below pole

$$r = \frac{(3/4)4}{1 - (3/4)\sin\theta} = \frac{12}{4 - 3\sin\theta}$$

37. $e = 2, x = 1, p = 1$

Vertical directrix to the right of the pole

$$r = \frac{2(1)}{1 + 2\cos\theta} = \frac{2}{1 + 2\cos\theta}$$

38. $e = \frac{3}{2}, x = -1, p = 1$

Vertical directrix to the left of the pole

$$r = \frac{3/2(1)}{1 - (3/2)\cos\theta} = \frac{3}{2 - 3\cos\theta}$$

39. Vertex: $\left(1, -\frac{\pi}{2}\right) \Rightarrow e = 1, p = 2$

Horizontal directrix below the pole

$$r = \frac{1(2)}{1 - 1\sin\theta} = \frac{2}{1 - \sin\theta}$$

40. Parabola, $e = 1$, vertex: $(8, 0)$

Vertical directrix to right of pole

$$r = \frac{ep}{1 + e\cos\theta} = \frac{16}{1 + \cos\theta}$$

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41. Vertex: $(5, \pi) \implies e = 1, p = 10$

Vertical directrix to left of pole

$$r = \frac{1(10)}{1 - 1\cos\theta} = \frac{10}{1 - \cos\theta}$$

42. Vertex: $\left(10, \frac{\pi}{2}\right) \implies e = 1, p = 20$

Horizontal directrix above pole

$$r = \frac{1(20)}{1 + 1\sin\theta} = \frac{20}{1 + \sin\theta}$$

43. Center: $(4, \pi), c = 4, a = 6, e = \frac{2}{3}$

Vertical directrix to the right of the pole

$$r = \frac{(2/3)p}{1 + (2/3)\cos\theta} = \frac{2p}{3 + 2\cos\theta}$$

$$2 = \frac{2p}{3 + 2\cos 0} = \frac{2p}{5} \implies p = 5$$

$$r = \frac{10}{3 + 2\cos\theta}$$

44. Center: $\left(1, \frac{3\pi}{2}\right), c = 1, a = 3, e = \frac{1}{3}$

Horizontal directrix above the pole

$$r = \frac{(1/3)p}{1 + (1/3)\sin\theta} = \frac{p}{3 + \sin\theta}$$

$$2 = \frac{p}{3 + \sin(\pi/2)}$$

$$p = 8$$

$$r = \frac{8}{3 + \sin\theta}$$

45. Center: $(8, 0), c = 8, a = 12, e = \frac{c}{a} = \frac{2}{3}$

Vertical directrix to left of pole

$$r = \frac{(2/3)p}{1 - (2/3)\cos\theta} = \frac{2p}{3 - 2\cos\theta}$$

$$20 = \frac{2p}{3 - 2} = 2p \implies p = 10$$

$$r = \frac{20}{3 - 2\cos\theta}$$

46. Center: $\left(5, \frac{3\pi}{2}\right), c = 5, a = 4, e = \frac{5}{4}$

Horizontal directrix below the pole

$$r = \frac{(5/4)p}{1 - (5/4)\sin\theta} = \frac{5p}{4 - 5\sin\theta}$$

$$1 = \frac{5p}{4 - 5\sin(3\pi/2)}$$

$$p = \frac{9}{5}$$

$$r = \frac{5(9/5)}{4 - 5\sin\theta} = \frac{9}{4 - 5\sin\theta}$$

47. Center: $\left(\frac{5}{2}, \frac{\pi}{2}\right), c = \frac{5}{2}, a = \frac{3}{2}, e = \frac{5}{3}$

Horizontal directrix above the pole

$$r = \frac{(5/3)p}{1 + (5/3)\sin\theta} = \frac{5p}{3 + 5\sin\theta}$$

Substitute the point $\left(1, \frac{-3\pi}{2}\right)$ rather than $\left(-1, \frac{3\pi}{2}\right)$ in order to get a directrix between the vertices.

$$1 = \frac{5p}{3 + 5\sin(-3\pi/2)}$$

$$p = \frac{8}{5}$$

$$r = \frac{5(8/5)}{3 + 5\sin\theta} = \frac{8}{3 + 5\sin\theta}$$

48. Center: $\left(\frac{5}{2}, \frac{\pi}{2}\right), c = \frac{5}{2}, a = \frac{3}{2}, e = \frac{c}{a} = \frac{5}{3}$

Horizontal directrix above the pole

$$r = \frac{(5/3)p}{1 + (5/3)\sin\theta} = \frac{5p}{3 + 5\sin\theta}$$

$$1 = \frac{5p}{3 + 5\sin(\pi/2)} \implies p = \frac{8}{5}$$

$$r = \frac{8}{3 + 5\sin\theta}$$

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49. When $\theta = 0$, $r = c + a = ea + a = a(1 + e)$.

Therefore,

$$a(1 + e) = \frac{ep}{1 - e\cos 0}$$

$$a(1 + e)(1 - e) = ep$$

$$a(1 - e^2) = ep.$$

Thus, $r = \dfrac{ep}{1 - e\cos\theta} = \dfrac{(1 - e^2)a}{1 - e\cos\theta}$.

50. Minimum distance occurs when $\theta = \pi$.

$$r = \frac{(1 - e^2)a}{1 - e\cos\pi} = \frac{(1 - e)(1 + e)a}{1 + e} = a(1 - e)$$

Maximum distance occurs when $\theta = 0$.

$$r = \frac{(1 - e^2)a}{1 - e\cos 0} = \frac{(1 - e)(1 + e)a}{1 - e} = a(1 + e)$$

51. $r = \dfrac{[1 - (0.0167)^2](92.956 \times 10^6)}{1 - 0.0167\cos\theta}$

$\approx \dfrac{9.2930 \times 10^7}{1 - 0.0167\cos\theta}$

Perihelion distance:

$r = 92.956 \times 10^6(1 - 0.0167) \approx 9.1404 \times 10^7$

Aphelion distance:

$r = 92.956 \times 10^6(1 + 0.0167) \approx 9.4508 \times 10^7$

52. $a = 35.983 \times 10^6$, $e = 0.2056$

$r = \dfrac{(1 - 0.2056^2)(35.983 \times 10^6)}{1 - 0.2056\cos\theta}$

$\approx \dfrac{3.4462 \times 10^7}{1 - 0.2056\cos\theta}$

Perihelion distance: $a(1 - e) \approx 2.8585 \times 10^7$ miles

Aphelion distance: $a(1 + e) \approx 4.3381 \times 10^7$ miles

53. $r = \dfrac{(1 - 0.0484^2)77.841 \times 10^7}{1 - 0.0484\cos\theta}$

$= \dfrac{7.7659 \times 10^8}{1 - 0.0484\cos\theta}$

Perihelion:

$r = 77.841 \times 10^7(1 - 0.0484) \approx 7.4073 \times 10^8$ km

Aphelion:

$r = 77.841 \times 10^7(1 + 0.0484) \approx 8.1609 \times 10^8$ km

54. $a = 142.673 \times 10^7$, $e = 0.0542$

$r = \dfrac{(1 - 0.0542^2)(142.673 \times 10^7)}{1 - 0.0542\cos\theta}$

$\approx \dfrac{1.4225 \times 10^9}{1 - 0.0542\cos\theta}$

Perihelion distance: $a(1 - e) \approx 1.3494 \times 10^9$ km

Aphelion distance: $a(1 + e) \approx 1.5041 \times 10^9$ km

55. $a = 4.498 \times 10^9$, $e = 0.0086$, Neptune

$a = 5.906 \times 10^9$, $e = 0.2488$, Pluto

(a) Neptune: $r = \dfrac{(1 - 0.0086^2)4.498 \times 10^9}{1 - 0.0086\cos\theta} = \dfrac{4.4977 \times 10^9}{1 - 0.0086\cos\theta}$

Pluto: $r = \dfrac{(1 - 0.2488^2)5.906 \times 10^9}{1 - 0.2488\cos\theta} = \dfrac{5.5404 \times 10^9}{1 - 0.2488\cos\theta}$

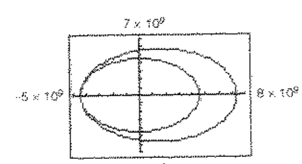
(b) Neptune: Perihelion: $4.498 \times 10^9(1 - 0.0086) \approx 4.4593 \times 10^9$ km

Aphelion: $4.498 \times 10^9(1 + 0.0086) \approx 4.5367 \times 10^9$ km

Pluto: Perihelion: $5.906 \times 10^9(1 - 0.2488) \approx 4.4366 \times 10^9$ km

Aphelion: $5.906 \times 10^9(1 + 0.2488) \approx 7.3754 \times 10^9$ km

(c)



(d) Yes. Pluto is closer to the sun for just a very short time. Pluto was considered the ninth planet because its mean distance from the sun is larger than that of Neptune.

(e) Although the graphs intersect, the orbits do not, and the planets won't collide.

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56. (a) Radius of earth $\approx$ 4000 miles. Choose $r = \dfrac{ep}{1 - e\cos\theta}$.

Vertices: $(126{,}800, 0)$ and $(4119, \pi)$

$$a = \frac{126{,}800 + 4119}{2} = 65{,}459.5$$

$$c = 65{,}459.5 - 4119 = 61{,}340.5$$

$$e = \frac{c}{a} = \frac{61{,}340.5}{65{,}459.5} \approx 0.937$$

$$2a = \frac{ep}{1 - e\cos 0} + \frac{ep}{1 - e\cos(\pi)} = \frac{ep}{1-e} + \frac{ep}{1+e} = \frac{2ep}{1-e^2}$$

Thus, $p = \dfrac{a(1-e^2)}{e} \approx 8525.2$. Thus, $r = \dfrac{ep}{1 - e\cos\theta} \approx \dfrac{7988.1}{1 - 0.937\cos\theta}$.

(b) When $\theta = 60°$, $r \approx 15{,}029$ and the distance from the surface of the earth to the satellite is $15{,}029 - 4000 = 11{,}029$ miles.

(c) When $\theta = 30°$, $r \approx 42{,}370$ and distance $= 38{,}370$ miles.

57. $r = \dfrac{4}{-3 - 3\sin\theta} = \dfrac{-4/3}{1 + \sin\theta}$

False. The directrix is below the pole.

58. $r^2 = \dfrac{16}{9 - 4\cos\left(\theta + \dfrac{\pi}{4}\right)}$

False. The graph is not an ellipse.

(It is two ellipses.)

59.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{r^2\cos^2\theta}{a^2} + \frac{r^2\sin^2\theta}{b^2} = 1$$

$$\frac{r^2\cos^2\theta}{a^2} + \frac{r^2(1 - \cos^2\theta)}{b^2} = 1$$

$$r^2b^2\cos^2\theta + r^2a^2 - r^2a^2\cos^2\theta = a^2b^2$$

$$r^2(b^2 - a^2)\cos^2\theta + r^2a^2 = a^2b^2$$

For an ellipse, $b^2 - a^2 = -c^2$. Hence,

$$-r^2c^2\cos^2\theta + r^2a^2 = a^2b^2$$

$$-r^2\left(\frac{c}{a}\right)^2\cos^2\theta + r^2 = b^2,\ e = \frac{c}{a}$$

$$-r^2e^2\cos^2\theta + r^2 = b^2$$

$$r^2(1 - e^2\cos^2\theta) = b^2$$

$$r^2 = \frac{b^2}{1 - e^2\cos^2\theta}.$$

60.

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\frac{r^2\cos^2\theta}{a^2} - \frac{r^2\sin^2\theta}{b^2} = 1$$

$$\frac{r^2\cos^2\theta}{a^2} - \frac{r^2(1 - \cos^2\theta)}{b^2} = 1$$

$$r^2b^2\cos^2\theta - r^2a^2 + r^2a^2\cos^2\theta = a^2b^2$$

$$r^2(b^2 + a^2)\cos^2\theta - r^2a^2 = a^2b^2$$

$$a^2 + b^2 = c^2$$

$$r^2c^2\cos^2\theta - r^2a^2 = a^2b^2$$

$$r^2\left(\frac{c}{a}\right)^2\cos^2\theta - r^2 = b^2,\ e = \frac{c}{a}$$

$$r^2e^2\cos^2\theta - r^2 = b^2$$

$$r^2(e^2\cos^2\theta - 1) = b^2$$

$$r^2 = \frac{b^2}{e^2\cos^2\theta - 1} = \frac{-b^2}{1 - e^2\cos^2\theta}$$

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61. $\dfrac{x^2}{169} + \dfrac{y^2}{144} = 1$

$a = 13, b = 12, c = 5, e = \dfrac{5}{13}$

$r^2 = \dfrac{144}{1 - (25/169)\cos^2\theta} = \dfrac{24{,}336}{169 - 25\cos^2\theta}$

62. $\dfrac{x^2}{9} - \dfrac{y^2}{16} = 1$

$a = 3, b = 4, c = 5, e = \dfrac{5}{3}$

$r^2 = \dfrac{-16}{1 - (25/9)\cos^2\theta} = \dfrac{144}{25\cos^2\theta - 9}$

63. $\dfrac{x^2}{25} + \dfrac{y^2}{16} = 1$

$a = 5, b = 4, c = 3, e = \dfrac{3}{5}$

$r^2 = \dfrac{b^2}{1 - e^2\cos^2\theta} = \dfrac{16}{1 - (9/25)\cos^2\theta} = \dfrac{400}{25 - 9\cos^2\theta}$

64. $\dfrac{x^2}{36} - \dfrac{y^2}{4} = 1$

$a = 6, b = 2, c = \sqrt{40} = 2\sqrt{10}, e = \dfrac{2\sqrt{10}}{6} = \dfrac{\sqrt{10}}{3}$

$r^2 = \dfrac{-b^2}{1 - e^2\cos^2\theta} = \dfrac{-4}{1 - (10/9)\cos^2\theta} = \dfrac{36}{10\cos^2\theta - 9}$

65. Center: $(x, y) = (0, 0), c = 5, a = 4, e = \dfrac{5}{4}$

$b^2 = c^2 - a^2 = 25 - 16 = 9 \Longrightarrow b = 3$

$r^2 = \dfrac{-b^2}{1 - e^2\cos^2\theta} = \dfrac{-9}{1 - (25/16)\cos^2\theta} = \dfrac{144}{25\cos^2\theta - 16}$

66. Center: $(x, y) = (0, 0), c = 4, a = 5, e = \dfrac{4}{5}$

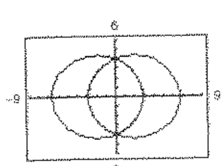
$b^2 = a^2 - c^2 = 25 - 16 = 9 \Longrightarrow b = 3$

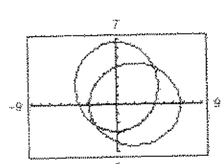
$r^2 = \dfrac{b^2}{1 - e^2\cos^2\theta} = \dfrac{9}{1 - (16/25)\cos^2\theta} = \dfrac{225}{25 - 16\cos^2\theta}$

67. $r = \dfrac{4}{1 - 0.4\cos\theta}$

Vertical directrix to left of pole

(a) $e = 0.4 \Longrightarrow$ ellipse





(b) $r = \dfrac{4}{1 + 0.4\cos\theta}$

Vertical directrix to right of pole

Graph is reflected in line $\theta = \pi/2$.

$r = \dfrac{4}{1 - 0.4\sin\theta}$

Horizontal directrix below pole

90° rotation counterclockwise

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68. The lengths of the major and minor axes increase as p increases.

Example: $r = \dfrac{(0.5)2}{1 + (0.5)\sin\theta}$

$r = \dfrac{(0.5)4}{1 + (0.5)\sin\theta}$

69. Answers will vary.

70.
$$r = a\sin\theta + b\cos\theta$$
$$r^2 = a\,r\sin\theta + b\,r\cos\theta$$
$$x^2 + y^2 = ay + bx$$

Circle

71. $4\sqrt{3}\tan\theta - 3 = 1$

$$\tan\theta = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$
$$\theta = \frac{\pi}{6} + n\pi$$

72. $6\cos x - 2 = 1$

$$\cos x = \frac{1}{2}$$
$$x = \frac{\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi$$

73. $12\sin^2\theta = 9$

$$\sin^2\theta = \frac{3}{4}$$
$$\sin\theta = \pm\frac{\sqrt{3}}{2}$$
$$\theta = \frac{\pi}{3} + n\pi, \frac{2\pi}{3} + n\pi$$

74. $9\csc^2 x - 10 = 2$

$$\csc^2 x = \frac{4}{3}$$
$$\sin^2 x = \frac{3}{4}$$
$$\sin x = \frac{\pm\sqrt{3}}{2}$$
$$x = \frac{\pi}{3} + n\pi, \frac{2\pi}{3} + n\pi$$

75. $2\cot x = 5\cos\dfrac{\pi}{2}$

$$2\cot x = 0$$
$$\cot x = 0$$
$$x = \frac{\pi}{2} + n\pi$$

76. $\sqrt{2}\sec\theta = 2\csc\dfrac{\pi}{4}$

$$\sqrt{2}\sec\theta = 2\sqrt{2}$$
$$\sec\theta = 2$$
$$\cos\theta = \frac{1}{2}$$
$$\theta = \frac{\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi$$

For Exercises 77–80: $\sin u = -\dfrac{3}{5}, \cos u = \dfrac{4}{5}, \cos v = \dfrac{1}{\sqrt{2}}, \sin v = -\dfrac{1}{\sqrt{2}}$

77. $\cos(u + v) = \cos u\cos v - \sin u\sin v$

$$= \frac{4}{5}\left(\frac{1}{\sqrt{2}}\right) - \left(-\frac{3}{5}\right)\left(-\frac{1}{\sqrt{2}}\right)$$
$$= \frac{1}{5\sqrt{2}} = \frac{\sqrt{2}}{10}$$

78. $\sin(u + v) = \sin u\cos v + \sin v\cos u$

$$= \left(\frac{-3}{5}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{-1}{\sqrt{2}}\right)\left(\frac{4}{5}\right)$$
$$= \frac{-7}{5\sqrt{2}} = \frac{-7\sqrt{2}}{10}$$

79. $\sin(u - v) = \sin u\cos v - \sin v\cos u$

$$= \left(-\frac{3}{5}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(-\frac{1}{\sqrt{2}}\right)\left(\frac{4}{5}\right)$$
$$= \frac{1}{5\sqrt{2}} = \frac{\sqrt{2}}{10}$$

80. $\cos(u - v) = \cos u\cos v + \sin u\sin v$

$$= \left(\frac{4}{5}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{-3}{5}\right)\left(\frac{-1}{\sqrt{2}}\right)$$
$$= \frac{7}{5\sqrt{2}} = \frac{7\sqrt{2}}{10}$$

81. ${}_{12}C_9 = 220$

82. ${}_{18}C_{16} = 153$

83. ${}_{10}P_3 = 720$

84. ${}_{29}P_2 = 812$

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Review Exercises for Chapter 9

1. Radius $= \sqrt{(-3-0)^2 + (-4-0)^2}$

$= \sqrt{9+16} = \sqrt{25} = 5$

$x^2 + y^2 = 25$

2. Radius $= \sqrt{(8-0)^2 + (-15-0)^2}$

$= \sqrt{64+225} = \sqrt{289} = 17$

$x^2 + y^2 = 289$

3. Radius $= \dfrac{1}{2}\sqrt{(5-(-1))^2 + (6-2)^2}$

$= \dfrac{1}{2}\sqrt{36+16} = \dfrac{1}{2}\sqrt{52} = \sqrt{13}$

Center $= \left(\dfrac{5+(-1)}{2}, \dfrac{6+2}{2}\right) = (2, 4)$

$(x-2)^2 + (y-4)^2 = 13$

4. Radius $= \dfrac{1}{2}\sqrt{(6-(-2))^2 + (-5-3)^2}$

$= \dfrac{1}{2}\sqrt{64+64} = 4\sqrt{2}$

Center $= \left(\dfrac{-2+6}{2}, \dfrac{3-5}{2}\right) = (2, -1)$

$(x-2)^2 + (y+1)^2 = 32$

5. $\dfrac{1}{2}x^2 + \dfrac{1}{2}y^2 = 18$

$x^2 + y^2 = 36$

Center: $(0, 0)$

Radius: 6

6. $\dfrac{3}{4}x^2 + \dfrac{3}{4}y^2 = 1$

$x^2 + y^2 = \dfrac{4}{3}$

Center: $(0, 0)$

Radius: $\dfrac{2}{\sqrt{3}} = \dfrac{2\sqrt{3}}{3}$

7.
$$16x^2 + 16y^2 - 16x + 24y - 3 = 0$$
$$16\left(x^2 - x + \tfrac{1}{4}\right) + 16\left(y^2 + \tfrac{3}{2}y + \tfrac{9}{16}\right) = 3 + 4 + 9$$
$$16\left(x - \tfrac{1}{2}\right)^2 + 16\left(y + \tfrac{3}{4}\right)^2 = 16$$
$$\left(x - \tfrac{1}{2}\right)^2 + \left(y + \tfrac{3}{4}\right)^2 = 1$$

Center: $\left(\tfrac{1}{2}, -\tfrac{3}{4}\right)$

Radius: 1

8.
$$4x^2 + 4y^2 + 32x - 24y + 51 = 0$$
$$4(x^2 + 8x + 16) + 4(y^2 - 6y + 9) = -51 + 64 + 36$$
$$4(x+4)^2 + 4(y-3)^2 = 49$$
$$(x+4)^2 + (y-3)^2 = \tfrac{49}{4}$$

Center: $(-4, 3)$

Radius: $\tfrac{7}{2}$

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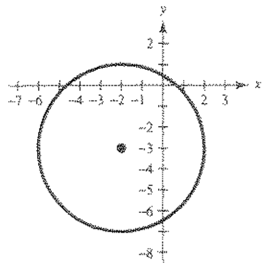
9. $(x^2 + 4x + 4) + (y^2 + 6y + 9) = 3 + 4 + 9$

$$(x + 2)^2 + (y + 3)^2 = 16$$

Center: $(-2, -3)$

Radius: 4

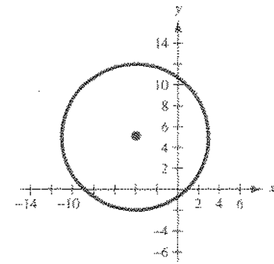


10. $(x^2 + 8x + 16) + (y^2 - 10y + 25) = 8 + 16 + 25$

$$(x + 4)^2 + (y - 5)^2 = 49$$

Center: $(-4, 5)$

Radius: 7



11. x-intercepts: $(x - 3)^2 + (0 + 1)^2 = 7$

$$(x - 3)^2 = 6$$

$$x - 3 = \pm\sqrt{6}$$

$$x = 3 \pm \sqrt{6}$$

$$\left(3 \pm \sqrt{6}, 0\right)$$

y-intercepts: $(0 - 3)^2 + (y + 1)^2 = 7$

$$(y + 1)^2 = -2, \text{ impossible}$$

No y-intercepts

12. x-intercepts: $(x + 5)^2 + (0 - 6)^2 = 27$

$$(x + 5)^2 = -9, \text{ impossible}$$

No x-intercepts

y-intercepts: $(0 + 5)^2 + (y - 6)^2 = 27$

$$(y - 6)^2 = 2$$

$$y - 6 = \pm\sqrt{2}$$

$$y = 6 \pm \sqrt{2}$$

$$\left(0, 6 \pm \sqrt{2}\right)$$

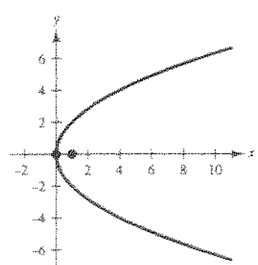
13. $4x - y^2 = 0$

$$y^2 = 4(1)x, \ p = 1$$

Vertex: $(0, 0)$

Focus: $(1, 0)$

Directrix: $x = -1$



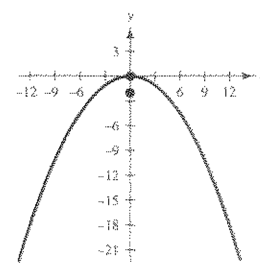
14. $y = -\frac{1}{8}x^2$

$$x^2 = 4(-2)y, \ p = -2$$

Vertex: $(0, 0)$

Focus: $(0, -2)$

Directrix: $y = 2$



15. $\frac{1}{2}y^2 + 18x = 0$

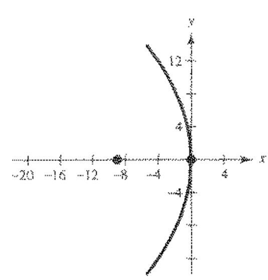
$$\frac{1}{2}y^2 = -18x$$

$$y^2 = -36x = 4(-9)x, \ p = -9$$

Vertex: $(0, 0)$

Focus: $(-9, 0)$

Directrix: $x = 9$



16. $\frac{1}{4}y - 8x^2 = 0$

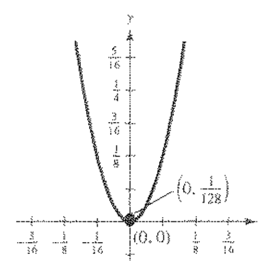
$$8x^2 = \frac{1}{4}y$$

$$x^2 = \frac{1}{32}y = 4\left(\frac{1}{128}\right)y, \ p = \frac{1}{128}$$

Vertex: $(0, 0)$

Focus: $\left(0, \frac{1}{128}\right)$

Directrix: $y = -\frac{1}{128}$



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17. Vertex: $(0, 0)$

Focus: $(-6, 0)$

Parabola opens to left.

$y^2 = 4px$

$y^2 = 4(-6)x$

$y^2 = -24x$

18. Vertex: $(4, 2)$

Focus: $(4, 0)$

Vertical axis, $p = -2$

$(x - 4)^2 = 4(-2)(y - 2)$

$(x - 4)^2 = -8(y - 2)$

19. Vertex: $(-6, 4)$

Passes through $(0, 0)$

Vertical axis

$(x + 6)^2 = 4p(y - 4)$

$(0 + 6)^2 = 4p(0 - 4)$

$36 = -16p$

$-\frac{9}{4} = p$

$(x + 6)^2 = 4\left(-\frac{9}{4}\right)(y - 4)$

$(x + 6)^2 = -9(y - 4)$

20. Vertex: $(0, 5)$

$(y - 5)^2 = 4p(x - 0) = 4px$

$(6, 0)$ on graph:

$(0 - 5)^2 = 4p(6) \implies p = \frac{25}{24}$.

$(y - 5)^2 = 4\left(\frac{25}{24}\right)x = \frac{25}{6}x$

21. $x^2 = -2y = 4\left(-\frac{1}{2}\right)y, p = -\frac{1}{2}$

Focus: $\left(0, -\frac{1}{2}\right)$

$d_1 = \frac{1}{2} + b$

$d_2 = \sqrt{(2 - 0)^2 + \left(-2 + \frac{1}{2}\right)^2} = \frac{5}{2}$

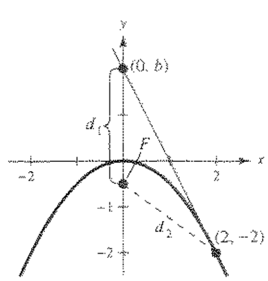
$d_1 = d_2 \implies \frac{1}{2} + b = \frac{5}{2} \implies b = 2$

Slope of tangent line: $\frac{b + 2}{0 - 2} = \frac{4}{-2} = -2$

Equation: $y + 2 = -2(x - 2)$

$y = -2x + 2$

x-intercept: $(1, 0)$



22. $-2x = y^2$

$4\left(-\frac{1}{2}x\right) = y^2$

$p = -\frac{1}{2}$

Focus: $\left(-\frac{1}{2}, 0\right)$

Let $(b, 0)$ be the x-intercept of the tangent line.

$d_1 = \frac{1}{2} + b$

$d_2 = \sqrt{\left(-8 + \frac{1}{2}\right)^2 + (-4 - 0)^2} = \frac{17}{2}$

$\frac{1}{2} + b = \frac{17}{2} \implies b = 8$

$m = \frac{-4 - 0}{-8 - 8} = \frac{1}{4}$

$y - 0 = \frac{1}{4}(x - 8)$

$y = \frac{1}{4}x - 2$

x-intercept: $(8, 0)$

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23. $x^2 = 4p(y - 12)$

$(4, 10)$ on curve:

$16 = 4p(10 - 12) = -8p \implies p = -2$

$x^2 = 4(-2)(y - 12) = -8y + 96$

$y = \dfrac{-x^2 + 96}{8}$

$y = 0$ if $x^2 = 96 \implies x = 4\sqrt{6} \implies$ width is $8\sqrt{6}$ meters.

24. (a) Parabola:

Vertex: $(0, 4)$

Passes through $(\pm 4, 0)$

$x^2 = 4p(y - 4)$

$16 = 4p(0 - 4)$

$16 = -16p$

$-1 = p$

$x^2 = -4(y - 4)$

Circle:

Passes through $(\pm 4, 0)$

Radius: $r = 8$

Center: $(0, k)$

$x^2 + (y - k)^2 = 8^2$

$(\pm 4)^2 + (0 - k)^2 = 8^2$

$16 + k^2 = 64$

$k^2 = 48$

$k = -\sqrt{48} = -4\sqrt{3}$

$x^2 + \left(y + 4\sqrt{3}\right)^2 = 64$

(b) Parabola:

$x^2 = -4(y - 4) \implies y = -\frac{1}{4}x^2 + 4$

Circle:

$x^2 + \left(y + 4\sqrt{3}\right)^2 = 64 \implies y = \sqrt{64 - x^2} - 4\sqrt{3}$

$d = \left(-\frac{1}{4}x^2 + 4\right) - \left(\sqrt{64 - x^2} - 4\sqrt{3}\right)$

$= -\frac{1}{4}x^2 - \sqrt{64 - x^2} + 4 + 4\sqrt{3}$

x	0	1	2	3	4
d	2.928	2.741	2.182	1.262	0

25. $\dfrac{x^2}{4} + \dfrac{y^2}{16} = 1$

$a = 4, b = 2, c = \sqrt{16 - 4} = \sqrt{12} = 2\sqrt{3}$

Center: $(0, 0)$

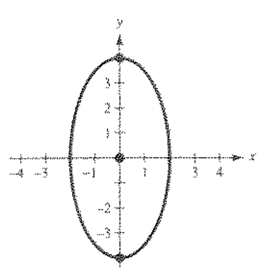
Vertices: $(0, \pm 4)$

Foci: $\left(0, \pm 2\sqrt{3}\right)$

Eccentricity $= \dfrac{c}{a}$

$= \dfrac{2\sqrt{3}}{4}$

$= \dfrac{\sqrt{3}}{2}$



26. $\dfrac{x^2}{9} + \dfrac{y^2}{8} = 1$

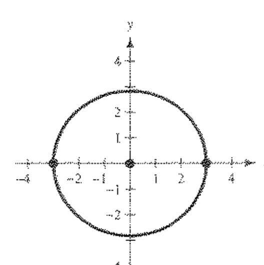
$a = 3, b = 2\sqrt{2}, c = \sqrt{9 - 8} = 1$

Center: $(0, 0)$

Vertices: $(\pm 3, 0)$

Foci: $(\pm 1, 0)$

Eccentricity $= \dfrac{c}{a} = \dfrac{1}{3}$



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27. $\dfrac{(x-4)^2}{6} + \dfrac{(y+4)^2}{9} = 1$

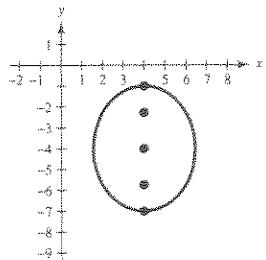
$a = 3, b = \sqrt{6}, c = \sqrt{9-6} = \sqrt{3}$

Center: $(4, -4)$

Vertices: $(4, -1), (4, -7)$

Foci: $\left(4, -4 \pm \sqrt{3}\right)$

Eccentricity $= \dfrac{c}{a} = \dfrac{\sqrt{3}}{3}$



28. $\dfrac{(x+1)^2}{16} + \dfrac{(y-3)^2}{6} = 1$

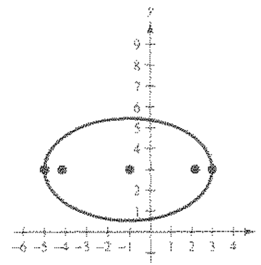
$a = 4, b = \sqrt{6}, c = \sqrt{16-6} = \sqrt{10}$

Center: $(-1, 3)$

Vertices: $(3, 3), (-5, 3)$

Foci: $\left(-1 \pm \sqrt{10}, 3\right)$

Eccentricity $= \dfrac{c}{a} = \dfrac{\sqrt{10}}{4}$



29. (a) $16(x^2 - 2x + 1) + 9(y^2 + 8y + 16) = -16 + 16 + 144$

$$16(x-1)^2 + 9(y+4)^2 = 144$$

$$\frac{(x-1)^2}{9} + \frac{(y+4)^2}{16} = 1$$

(b) Center: $(1, -4)$

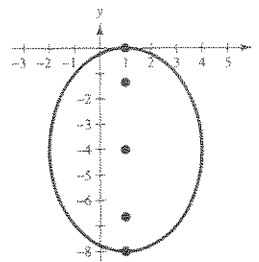
$a = 4, b = 3, c = \sqrt{16-9} = \sqrt{7}$

Vertices: $(1, 0), (1, -8)$

Foci: $\left(1, -4 \pm \sqrt{7}\right)$

$e = \dfrac{c}{a} = \dfrac{\sqrt{7}}{4}$

(c)



30. (a) $4(x^2 + 4x + 4) + 25(y^2 - 6y + 9) = -141 + 16 + 225$

$$4(x+2)^2 + 25(y-3)^2 = 100$$

$$\frac{(x+2)^2}{25} + \frac{(y-3)^2}{4} = 1$$

(b) $a = 5, b = 2, c = \sqrt{21}$

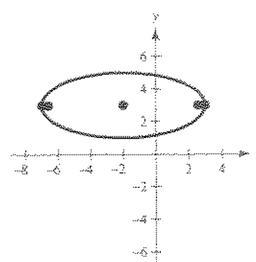
Center: $(-2, 3)$

Vertices: $(3, 3), (-7, 3)$

Foci: $\left(-2 \pm \sqrt{21}, 3\right)$

$e = \dfrac{\sqrt{21}}{5}$

(c)



31. (a) $3(x^2 + 4x + 4) + 8(y^2 - 14y + 49) = -403 + 12 + 392$

$$3(x+2)^2 + 8(y-7)^2 = 1$$

$$\frac{(x+2)^2}{1/3} + \frac{(y-7)^2}{1/8} = 1$$

—CONTINUED—

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31. —CONTINUED—

(b) Center: $(-2, 7)$

$$a = \frac{\sqrt{3}}{3}, b = \frac{\sqrt{2}}{4}$$

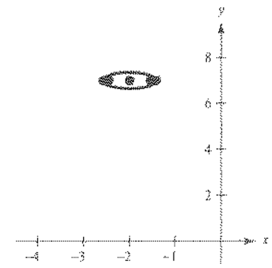
$$c^2 = a^2 - b^2 = \frac{1}{3} - \frac{1}{8} = \frac{5}{24} \implies c = \frac{\sqrt{30}}{12}$$

Vertices: $\left(-2 \pm \frac{\sqrt{3}}{3}, 7\right)$

Foci: $\left(-2 \pm \frac{\sqrt{30}}{12}, 7\right)$

Eccentricity: $\frac{c}{a} = \frac{\sqrt{30}/12}{\sqrt{3}/3} = \frac{\sqrt{10}}{4}$

(c)



32. (a)

$$x^2 + 20y^2 - 5x + 120y + 185 = 0$$

$$\left(x^2 - 5x + \frac{25}{4}\right) + 20(y^2 + 6y + 9) = -185 + \frac{25}{4} + 180$$

$$\left(x - \frac{5}{2}\right)^2 + 20(y + 3)^2 = \frac{5}{4}$$

$$\frac{[x - (5/2)]^2}{(5/4)} + \frac{(y + 3)^2}{(1/16)} = 1$$

(b) $a = \frac{\sqrt{5}}{2}, b = \frac{1}{4}, c = \sqrt{\frac{5}{4} - \frac{1}{16}} = \frac{\sqrt{19}}{4}$

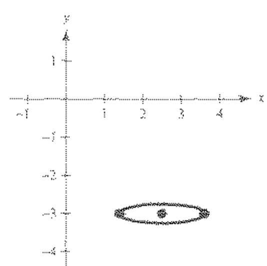
Center: $\left(\frac{5}{2}, -3\right)$

Vertices: $\left(\frac{5}{2} \pm \frac{\sqrt{5}}{2}, -3\right)$

Foci: $\left(\frac{5}{2} \pm \frac{\sqrt{19}}{4}, -3\right)$

$$e = \frac{c}{a} = \frac{\sqrt{19}/4}{\sqrt{5}/2} = \frac{\sqrt{95}}{10}$$

(c)



33. Vertices: $(\pm 5, 0)$

Foci: $(\pm 4, 0)$

$a = 5, c = 4 \implies b = 3$

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

34. Vertices: $(0, \pm 6)$

Passes through $(2, 2)$

Vertical major axis

Center: $(0, 0)$, $a = 6$

$$\frac{x^2}{b^2} + \frac{y^2}{36} = 1$$

$$\frac{2^2}{b^2} + \frac{2^2}{36} = 1$$

$$\frac{4}{b^2} = 1 - \frac{1}{9} = \frac{8}{9}$$

$$b^2 = \frac{36}{8} = \frac{9}{2}$$

$$\frac{x^2}{9/2} + \frac{y^2}{36} = 1$$

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35. Vertices: $(-3, 0), (7, 0)$

Foci: $(0, 0), (4, 0)$

Horizontal major axis

Center: $(2, 0)$

$a = 5, c = 2,$

$b = \sqrt{25 - 4} = \sqrt{21}$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$\frac{(x-2)^2}{25} + \frac{y^2}{21} = 1$$

36. Vertices: $(2, 0), (2, 4)$

Foci: $(2, 1), (2, 3)$

Vertical major axis

Center: $(2, 2)$

$a = 2, c = 1,$

$b = \sqrt{4 - 1} = \sqrt{3}$

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

$$\frac{(x-2)^2}{3} + \frac{(y-2)^2}{4} = 1$$

37. $a = 5, b = 4, c = \sqrt{a^2 - b^2} = \sqrt{25 - 16} = 3$

The foci should be placed 3 feet on either side of the center and have the same height as the pillars.

38. $\dfrac{x^2}{324} + \dfrac{y^2}{196} = 1, a = \sqrt{324} = 18, b = \sqrt{196} = 14$

Longest distance: $2a = 2(18) = 36$ feet

Shortest distance: $2b = 2(14) = 28$ feet

$c^2 = a^2 - b^2 = 128$

Foci: $(\pm 8\sqrt{2}, 0)$

Distance between foci: $16\sqrt{2} \approx 22.63$ feet

39. $a - c = 1.3495 \times 10^9$

$a + c = 1.5045 \times 10^9$

Adding, $2a = 2.854 \times 10^9 \implies a = 1.427 \times 10^9$. Then

$c = 1.5045 \times 10^9 - 1.427 \times 10^9 = 0.0775 \times 10^9$

$e = \dfrac{c}{a} \approx 0.0543.$

40. $a = \dfrac{72}{2} = 36$

$e = \dfrac{c}{a} = 0.2056 \implies c = ae = 7.4016$

$b^2 = a^2 - c^2 = 36^2 - 7.4016^2 \approx 1241.2$

$$\frac{x^2}{1296} + \frac{y^2}{1241.2} = 1$$

41. (a) $5y^2 - 4x^2 = 20$

$$\frac{y^2}{4} - \frac{x^2}{5} = 1$$

(b) $a = 2, b = \sqrt{5},$

$c = \sqrt{4 + 5} = 3$

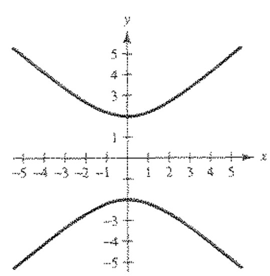
Center: $(0, 0)$

Vertices: $(0, \pm 2)$

Foci: $(0, \pm 3)$

Eccentricity $= \dfrac{c}{a} = \dfrac{3}{2}$

(c)



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42. (a) $x^2 - y^2 = \dfrac{9}{4}$

$$\frac{x^2}{(9/4)} - \frac{y^2}{(9/4)} = 1$$

(b) $a = \dfrac{3}{2}, b = \dfrac{3}{2},$

$$c = \sqrt{\frac{9}{4} + \frac{9}{4}}$$

$$= \sqrt{\frac{9}{2}} = \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$$

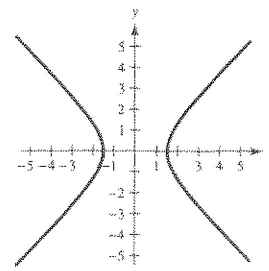
Center: $(0, 0)$

Vertices: $\left(\pm\dfrac{3}{2}, 0\right)$

Foci: $\left(\pm\dfrac{3\sqrt{2}}{2}, 0\right)$

Eccentricity $= \dfrac{c}{a} = \dfrac{3\sqrt{2}/2}{3/2} = \sqrt{2}$

(c)



43. (a) $9(x^2 - 2x + 1) - 16(y^2 + 2y + 1) = 151 + 9 - 16$

$$9(x - 1)^2 - 16(y + 1)^2 = 144$$

$$\frac{(x - 1)^2}{16} - \frac{(y + 1)^2}{9} = 1$$

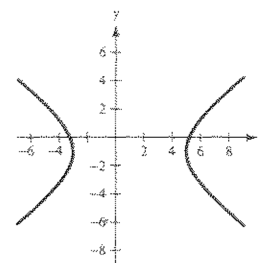
(b) Center: $(1, -1)$, $a = 4$, $b = 3$, $c = 5$

Vertices: $(5, -1), (-3, -1)$

Foci: $(6, -1), (-4, -1)$

Eccentricity: $\dfrac{5}{4}$

(c)



44. (a) $25(y^2 + 6y + 9) - 4(x^2 + 2x + 1) = -121 + 225 - 4$

$$25(y + 3)^2 - 4(x + 1)^2 = 100$$

$$\frac{(y + 3)^2}{4} - \frac{(x + 1)^2}{25} = 1$$

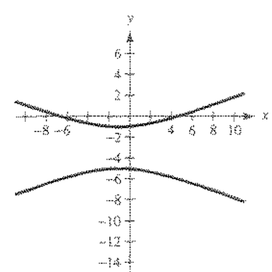
(b) Center: $(-1, -3)$, $a = 2$, $b = 5$, $c = \sqrt{29}$

Vertices: $(-1, -1), (-1, -5)$

Foci: $\left(-1, -3 \pm \sqrt{29}\right)$

Eccentricity: $\dfrac{\sqrt{29}}{2}$

(c)



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45. (a) $(y^2 - 2y + 1) - 4(x^2 + 12x + 36) = -59 + 1 - 144$

$$(y - 1)^2 - 4(x + 6)^2 = -202$$

$$\frac{(x + 6)^2}{(101/2)} - \frac{(y - 1)^2}{202} = 1$$

(b) Center: $(-6, 1)$

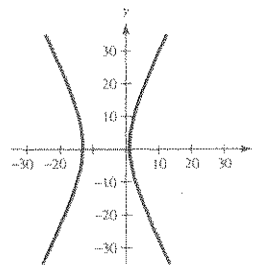
$a^2 = \frac{101}{2}, b^2 = 202, c^2 = \frac{101}{2} + 202 = \frac{505}{2}$

Vertices: $\left(-6 \pm \sqrt{\frac{101}{2}}, 1\right)$

Foci: $\left(-6 \pm \sqrt{\frac{505}{2}}, 1\right)$

Eccentricity: $e = \frac{c}{a} = \frac{\sqrt{505}}{\sqrt{101}} = \sqrt{5}$

(c)



46. (a) $9(x^2 - 8x + 16) - (y^2 - 8y + 16) = -119 + 144 - 16 = 9$

$$9(x - 4)^2 - (y - 4)^2 = 9$$

$$(x - 4)^2 - \frac{(y - 4)^2}{9} = 1$$

(b) Center: $(4, 4)$

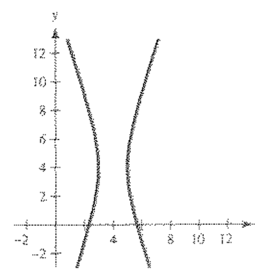
$a = 1, b = 3, c = \sqrt{10}$

Vertices: $(4 \pm 1, 4)$: $(3, 4), (5, 4)$

Foci: $\left(4 \pm \sqrt{10}, 4\right)$

Eccentricity: $e = \sqrt{10}$

(c)



47. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$a = 4$

$c^2 = a^2 + b^2 \implies 36 = 16 + b^2$

$\implies b = \sqrt{20} = 2\sqrt{5}$

$\frac{x^2}{16} - \frac{y^2}{20} = 1$

48. Vertices: $(0, \pm 1)$

Foci: $(0, \pm 3)$

Vertical transverse axis

Center: $(0, 0)$

$a = 1, c = 3, b = \sqrt{9 - 1} = \sqrt{8}$

$\frac{y^2}{1} - \frac{x^2}{8} = 1$

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49. Foci: $(0, 0), (8, 0) \implies c = 4$

Center: $(4, 0)$

Asymptotes:

$y = \pm 2(x - 4) \implies \dfrac{b}{a} = 2 \implies b = 2a$

$c^2 = a^2 + b^2$

$16 = a^2 + (2a)^2 = 5a^2 \implies a = \dfrac{4}{\sqrt{5}}, b = \dfrac{8}{\sqrt{5}}$

$$\frac{(x-4)^2}{16/5} - \frac{y^2}{64/5} = 1$$

50. Vertical transverse axis

Center: $(3, 0) \implies c = 2$

$\dfrac{a}{b} = 2 \implies a = 2b$

$$a^2 + b^2 = c^2$$
$$(2b)^2 + b^2 = 4$$
$$b^2 = \frac{4}{5}$$
$$a^2 = \frac{16}{5}$$
$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$
$$\frac{5y^2}{16} - \frac{5(x-3)^2}{4} = 1$$

51. $d_2 - d_1 = 186{,}000(0.0005)$

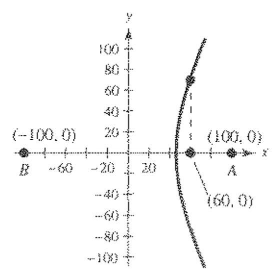
$2a = 93$

$a = 46.5$

$c = 100$

$b = \sqrt{c^2 - a^2}$

$\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$



$$x = 60 \implies y^2 = b^2\left(\frac{x^2}{a^2} - 1\right) = (100^2 - 46.5^2)\left(\frac{60^2}{46.5^2} - 1\right) \approx 5211.57 \implies y \approx 72.2$$

72.2 miles north

52. Let the friends be at B and C, you at the origin A. The sound at C is heard 2 seconds after B: $2a = CD - BD = 2\left(\frac{1100}{5280}\right) = \frac{5}{12}$. Thus, $a = \frac{5}{24}$, $c = 2$ and $b^2 = c^2 - a^2 = \frac{2279}{576}$. Thus, using miles, the hyperbola is

$$\frac{x^2}{(25/576)} - \frac{y^2}{(2279/576)} = 1.$$

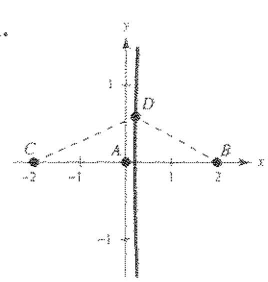
Now place the center at $(1, 0)$ and determine the second hyperbola.

$2a = DB - AD = 6\left(\dfrac{1100}{5280}\right) \implies a = \dfrac{5}{8}$

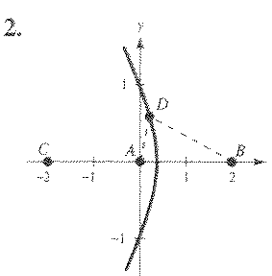
$c = 1$ and $b^2 = 1 - \dfrac{25}{64} = \dfrac{39}{64}$

$$\frac{(x-1)^2}{(25/64)} - \frac{y^2}{(39/64)} = 1$$

1.



2.



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53. $$3x^2 + 2y^2 - 12x + 12y + 29 = 0$$
$$3(x^2 - 4x + 4) + 2(y^2 + 6y + 9) = -29 + 12 + 18$$
$$3(x - 2)^2 + 2(y + 3)^2 = 1$$

Ellipse

54. $4x^2 + 4y^2 - 4x + 8y - 11 = 0$

$A = C = 4 \implies$ Circle

55. $$5x^2 - 2y^2 + 10x - 4y + 17 = 0$$
$$5(x^2 + 2x + 1) - 2(y^2 + 2y + 1) = -17 + 5 - 2$$
$$5(x + 1)^2 - 2(y + 1)^2 = -14$$
$$\frac{(y + 1)^2}{7} - \frac{(x + 1)^2}{(14/5)} = 1$$

Hyperbola

56. $-4y^2 + 5x + 3y + 7 = 0$

$A = 0, C = -4, AC = 0$

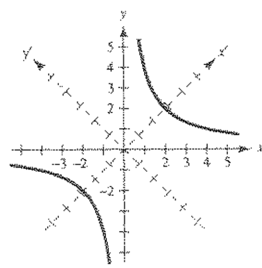
Parabola

57. $xy - 4 = 0$

$A = 0, B = 1, C = 0$

$\cot 2\theta = \dfrac{A - C}{B} = 0 \implies \theta = \dfrac{\pi}{4}$

$x = \dfrac{\sqrt{2}}{2}(x' - y'), y = \dfrac{\sqrt{2}}{2}(x' + y')$



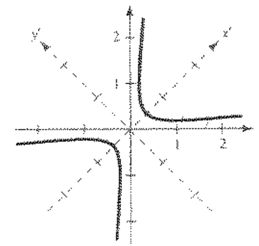
$$xy = 4$$
$$\frac{\sqrt{2}}{2}(x' - y')\frac{\sqrt{2}}{2}(x' + y') = 4$$
$$\frac{1}{2}(x')^2 - \frac{1}{2}(y')^2 = 4$$
$$\frac{(x')^2}{8} - \frac{(y')^2}{8} = 1$$

Hyperbola

58. $x^2 - 10xy + y^2 + 1 = 0$

$A = C = 1 \implies \cot 2\theta = 0 \implies \theta = \dfrac{\pi}{4}$

$x = \dfrac{\sqrt{2}}{2}(x' - y'), y = \dfrac{\sqrt{2}}{2}(x' + y')$



$$\left[\frac{\sqrt{2}}{2}(x' - y')\right]^2 - 10\left[\frac{\sqrt{2}}{2}(x' - y')\right]\left[\frac{\sqrt{2}}{2}(x' + y')\right] + \left[\frac{\sqrt{2}}{2}(x' + y')\right]^2 + 1 = 0$$
$$\frac{1}{2}(x')^2 + \frac{1}{2}(y')^2 - x'y' - 5((x')^2 - (y')^2) + \frac{1}{2}(x')^2 + x'y' + \frac{1}{2}(y')^2 = -1$$
$$-4(x')^2 + 6(y')^2 = -1$$
$$\frac{(x')^2}{1/4} - \frac{(y')^2}{1/6} = 1$$

Hyperbola

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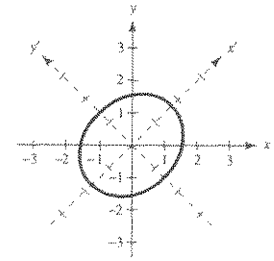
59. $5x^2 - 2xy + 5y^2 - 12 = 0$



$A = 5, B = -2, C = 5$

$\cot 2\theta = 0 \implies \theta = \dfrac{\pi}{4}$

$x = \dfrac{\sqrt{2}}{2}(x' - y'), y = \dfrac{\sqrt{2}}{2}(x' + y')$

$$5x^2 - 2xy + 5y^2 = 12$$

$$5\left[\frac{\sqrt{2}}{2}(x' - y')\right]^2 - 2\left[\frac{\sqrt{2}}{2}(x' - y')\right]\left[\frac{\sqrt{2}}{2}(x' + y')\right] + 5\left[\frac{\sqrt{2}}{2}(x' + y')\right]^2 = 12$$

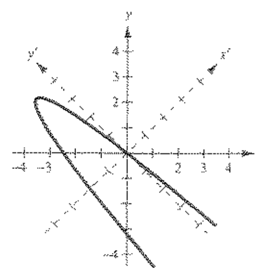
$$5\left[\frac{1}{2}(x')^2 - x'y' + \frac{1}{2}(y')^2\right] - (x')^2 + (y')^2 + 5\left[\frac{1}{2}(x')^2 + x'y' + \frac{1}{2}(y')^2\right] = 12$$

$$4(x')^2 + 6(y')^2 = 12$$

$$\frac{(x')^2}{3} + \frac{(y')^2}{2} = 1$$

Ellipse

60. $\cot 2\theta = \dfrac{4 - 4}{8} = 0 \implies \theta = \dfrac{\pi}{4}$



$x = \dfrac{\sqrt{2}}{2}(x' - y'), y = \dfrac{\sqrt{2}}{2}(x' + y')$

$$4\left[\frac{\sqrt{2}}{2}(x' - y')\right]^2 + 8\left[\frac{\sqrt{2}}{2}(x' - y')\frac{\sqrt{2}}{2}(x' + y')\right] + 4\left[\frac{\sqrt{2}}{2}(x' + y')\right]^2$$

$$+ 7\sqrt{2}\left[\frac{\sqrt{2}}{2}(x' - y')\right] + 9\sqrt{2}\left[\frac{\sqrt{2}}{2}(x' + y')\right] = 0$$

$$2[(x')^2 + (y')^2 - 2x'y'] + 4[(x')^2 - (y')^2] + 2[(x')^2 + (y')^2 + 2x'y'] + 7(x' - y') + 9(x' + y') = 0$$

$$8(x')^2 + 16x' + 2y' = 0$$

$$y' = -4(x')^2 - 8x'$$

Parabola

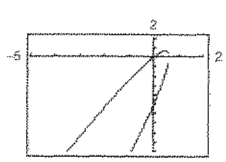
61. (a) $B^2 - 4AC = (-8)^2 - 4(16)(1) = 0$

Parabola

(b) $y^2 + (5 - 8x)y + (16x^2 - 10x) = 0$

$$y = \frac{(8x - 5) \pm \sqrt{(5 - 8x)^2 - 4(16x^2 - 10x)}}{2}$$

(c)

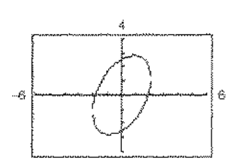


62. (a) $B^2 - 4AC = -300 \implies$ Ellipse

(b) $7y^2 - 8xy + (13x^2 - 45) = 0$

$$y = \frac{8x \pm \sqrt{(64x^2) - 4(7)(13x^2 - 45)}}{14}$$

(c)



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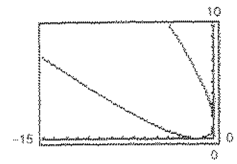
63. (a) $B^2 - 4AC = (2)^2 - 4(1)(1) = 0$

Parabola

(b) $y^2 + (2x - 2\sqrt{2})y + (x^2 + 2\sqrt{2}x + 2) = 0$

$$y = \frac{(2\sqrt{2} - 2x) \pm \sqrt{(2x - 2\sqrt{2})^2 - 4(x^2 + 2\sqrt{2}x + 2)}}{2}$$

(c)



64. (a) $B^2 - 4AC = 100 - 4$

$= 96 > 0 \implies$ Hyperbola

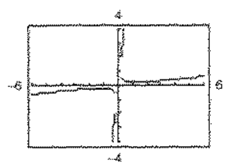
(b) $y^2 - 10xy + (x^2 + 1) = 0$

$$y = \frac{10x \pm \sqrt{100x^2 - 4(x^2 + 1)}}{2}$$

$$y = \frac{10x \pm \sqrt{96x^2 - 4}}{2}$$

$$y = 5x \pm \sqrt{24x^2 - 1}$$

(c)



65. Adding the equations,

$24x + 240 = 0 \implies x = -10$. Then:

$4(100) + y^2 - 560 - 24y + 304 = 0$

$y^2 - 24y + 144 = 0$

$(y - 12)^2 = 0 \implies y = 12$

Solution: $(-10, 12)$

66. $4x^2 + 4y^2 = 100$

$9x - 4y^2 = 0$

Adding:

$4x^2 + 9x = 100$

$4x^2 + 9x - 100 = 0$

$(x - 4)(4x + 25) = 0 \implies x = 4, -\frac{25}{4}$

If $x = 4$:

$4y^2 = 9(4) = 36$

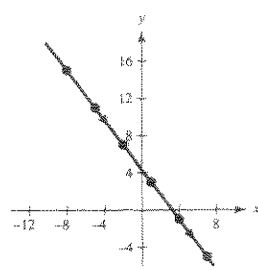
$y^2 = 9 \implies y = \pm 3$

If $x = -\frac{25}{4}$, $9\left(-\frac{25}{4}\right) = 4y^2$, impossible.

Answer: $(4, 3), (4, -3)$

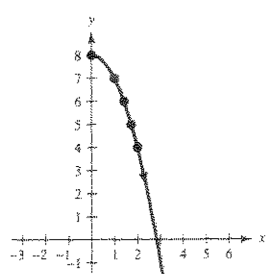
67.

t	-2	-1	0	1	2	3
x	-8	-5	-2	1	4	7
y	15	11	7	3	-1	-5



68. $x = \sqrt{t}, y = 8 - t$

t	0	1	2	3	4
x	0	1	$\sqrt{2}$	$\sqrt{3}$	2
y	8	7	6	5	4



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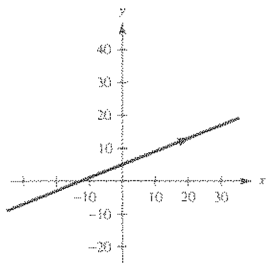
69. $x = 5t - 1,\ y = 2t + 5$

$t = \frac{1}{5}(x + 1) \implies$

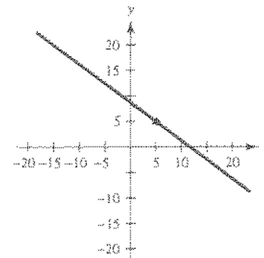
$y = \frac{2}{5}(x + 1) + 5 = \frac{2}{5}x + \frac{27}{5}$, line



70. $x = 4t + 1,\ y = 8 - 3t$

$t = \frac{1}{4}(x - 1) \implies$

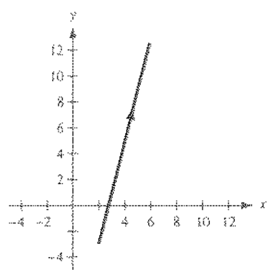
$y = 8 - 3\left(\frac{1}{4}(x - 1)\right) = -\frac{3}{4}x + \frac{35}{4}$



71. $x = t^2 + 2,\ y = 4t^2 - 3$

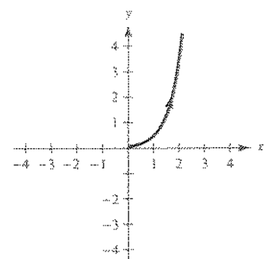
$t^2 = x - 2 \implies$

$y = 4(x - 2) - 3 = 4x - 11, \quad x \geq 2$



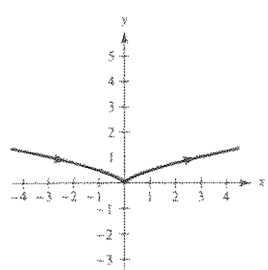
72. $x = \ln 4t,\ y = t^2$

$e^x = 4t,\ t = \frac{1}{4}e^x \implies y = \left(\frac{1}{4}e^x\right)^2 = \frac{1}{16}e^{2x}$



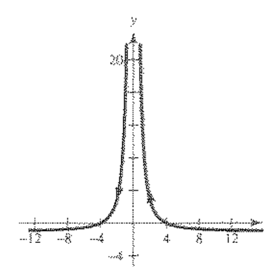
73. $x = t^3,\ y = \dfrac{1}{2}t^2$

$t = x^{1/3} \implies y = \dfrac{1}{2}x^{2/3}$



74. $x = \dfrac{4}{t},\ y = t^2 - 1$

$t = \dfrac{4}{x} \implies y = \dfrac{16}{x^2} - 1$

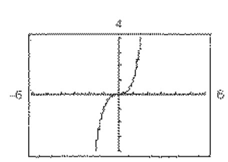


75. $x = \sqrt[3]{t}$

$y = t$

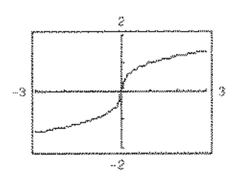
$t = x^3 \implies y = t = x^3$

$y = x^3$



76. $x = t$

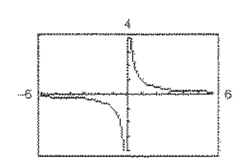
$y = \sqrt[3]{t} = \sqrt[3]{x} = x^{1/3}$



77. $x = \dfrac{1}{t}$

$y = t$

$y = t = \dfrac{1}{x}$



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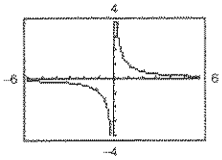
78. $x = t$

$y = \frac{1}{t} = \frac{1}{x}$

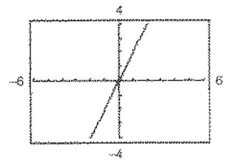


79. $x = 2t$

$y = 4t$

$y = 2(2t) = 2x$

Line

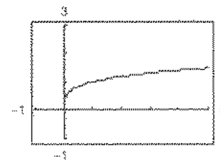


80. $x = t^2,\ y = \sqrt{t}$

$t = y^2$

$x = (y^2)^2 = y^4,\ y \geq 0$

$y = \sqrt[4]{x},\ x \geq 0$



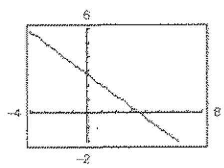
81. $x = 1 + 4t$

$y = 2 - 3t$

$t = \frac{x - 1}{4} \Rightarrow y = 2 - 3\left(\frac{x - 1}{4}\right) = 2 - \frac{3}{4}x + \frac{3}{4}$

$y = \frac{11}{4} - \frac{3}{4}x$

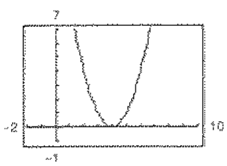
$3x + 4y - 11 = 0$



82. $x = t + 4,\ y = t^2$

$t = x - 4$

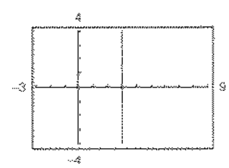
$y = (x - 4)^2$



83. $x = 3$

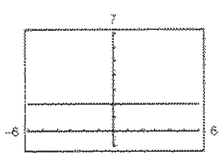
$y = t$

Vertical line: $x = 3$



84. $x = t$

$y = 2$

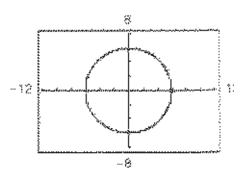


85. $x = 6 \cos \theta, y = 6 \sin \theta$

$\cos \theta = \frac{x}{6}, \sin \theta = \frac{y}{6}$

$\frac{x^2}{36} + \frac{y^2}{36} = 1$

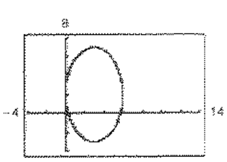
$x^2 + y^2 = 36$



86. $x = 3 + 3 \cos \theta, y = 2 + 5 \sin \theta$

$\cos \theta = \frac{x - 3}{3}, \sin \theta = \frac{y - 2}{5}$

$\frac{(x - 3)^2}{9} + \frac{(y - 2)^2}{25} = 1$



87. $y = 6x + 2$

$x = t, y = 6t + 2$

$x = -t, y = -6t + 2$

Other answers possible

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88. $x = t,\ y = 10 - t$

$x = -t,\ y = 10 + t$

Many answers possible

89. $y = x^2 + 2$

$x = t, y = t^2 + 2$

$x = t + 1, y = (t + 1)^2 + 2 = t^2 + 2t + 3$

Other answers possible

90. $x = t,\ y = 2t^3 + 5t$

$x = -t,\ y = -2t^3 - 5t$

Many answers possible

91. $x = x_1 + t(x_2 - x_1) = 3 + t(8 - 3) = 5t + 3$

$y = y_1 + t(y_2 - y_1) = 5 + t(0 - 0) = 5$

or $x = t,\ y = 5$

92. $x = x_1 + t(x_2 - x_1) = 2 + t(2 - 2) = 2$

$y = y_1 + t(y_2 - y_1)$

$= -1 + t(4 - (-1)) = -1 + 5t$

or $x = 2,\ y = t$

93. $x = x_1 + t(x_2 - x_1)$

$= -1 + t[10 - (-1)] = 11t - 1$

$y = y_1 + t(y_2 - y_1) = 6 + t(0 - 6) = -6t + 6$

94. $x = x_1 + t(x_2 - x_1) = 0 + t\left(\frac{5}{2} - 0\right) = \frac{5}{2}t$

$y = y_1 + t(y_2 - y_1) = 0 + t(6 - 0) = 6t$

or $x = 5t,\ y = 12t$

95. (90, 4) is on the curve:

$$90 = 0.82v_0t \implies v_0 = \frac{90}{0.82t}$$

$$4 = 7 + 0.57\left[\frac{90}{0.82t}\right]t - 16t^2 \implies$$

$$16t^2 = 3 + \frac{0.57(90)}{0.82} \implies t \approx 2.024$$

$$\text{Hence, } v_0 \approx \frac{90}{0.82(2.024)} \approx 54.23 \text{ ft/sec.}$$

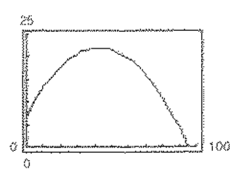
96. From Exercise 95, $v_0 = 54.23$.

$x = 0.82(54.23)t = 44.47t$

$y = 7 + 0.57(54.23)t - 16t^2$

$= 7 + 30.91t - 16t^2$

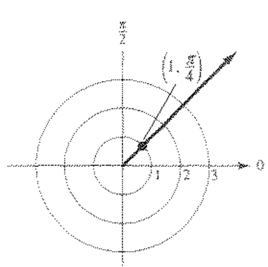
97. From Exercise 96:



The maximum height is approximately 21.9 feet for $t \approx 0.97$.

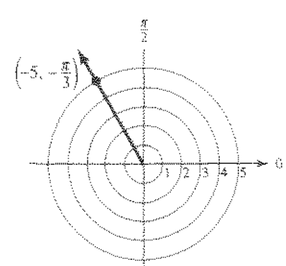
98. From Exercise 95, $t \approx 2.024$ seconds.

99. $\left(1, \frac{\pi}{4}\right)$



$\left(1, -\frac{7\pi}{4}\right), \left(-1, \frac{5\pi}{4}\right), \left(-1, -\frac{3\pi}{4}\right)$

100.



$\left(-5, \frac{5\pi}{3}\right), \left(5, \frac{2\pi}{3}\right), \left(5, -\frac{4\pi}{3}\right)$

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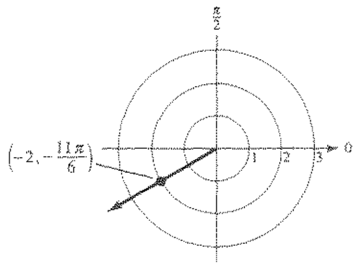
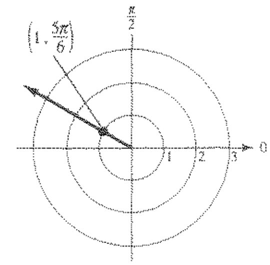
101. $(r, \theta) = \left(-2, -\dfrac{11\pi}{6}\right)$



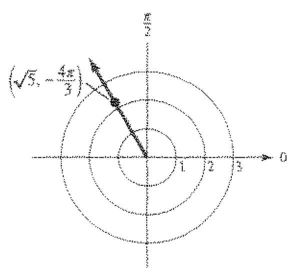
$\left(-2, \dfrac{\pi}{6}\right), \left(2, \dfrac{7\pi}{6}\right), \left(2, -\dfrac{5\pi}{6}\right)$

102. $(r, \theta) = \left(1, \dfrac{5\pi}{6}\right)$



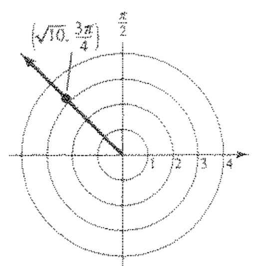
$\left(-1, -\dfrac{\pi}{6}\right), \left(-1, \dfrac{11\pi}{6}\right), \left(1, -\dfrac{7\pi}{6}\right)$

103. $\left(\sqrt{5}, -\dfrac{4\pi}{3}\right)$



$\left(\sqrt{5}, \dfrac{2\pi}{3}\right), \left(-\sqrt{5}, -\dfrac{\pi}{3}\right), \left(-\sqrt{5}, \dfrac{5\pi}{3}\right)$

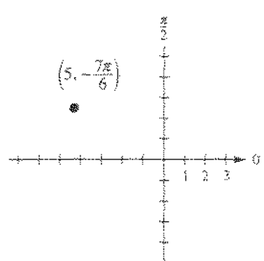
104.



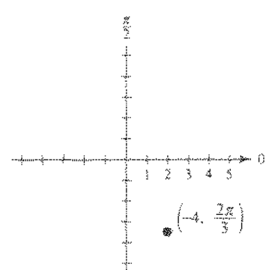
$(r, \theta) = \left(\sqrt{10}, -\dfrac{5\pi}{4}\right), \left(-\sqrt{10}, -\dfrac{\pi}{4}\right), \left(-\sqrt{10}, \dfrac{7\pi}{4}\right)$

105. $(r, \theta) = \left(5, -\dfrac{7\pi}{6}\right)$

$(x, y) = \left(-\dfrac{5\sqrt{3}}{2}, \dfrac{5}{2}\right)$



106.



$(r, \theta) = \left(-4, \dfrac{2\pi}{3}\right)$

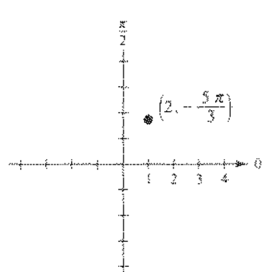
$(x, y) = \left(-4 \cos \dfrac{2\pi}{3}, -4 \sin \dfrac{2\pi}{3}\right) = \left(2, -2\sqrt{3}\right)$

107. $\left(2, -\dfrac{5\pi}{3}\right)$

$x = r \cos \theta = 2\left(\dfrac{1}{2}\right) = 1$

$y = r \sin \theta = 2\left(\dfrac{\sqrt{3}}{2}\right) = \sqrt{3}$

$(x, y) = \left(1, \sqrt{3}\right)$



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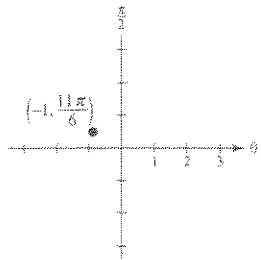
108. $\left(-1, \frac{11\pi}{6}\right)$

$x = r\cos\theta = -1\left(\frac{\sqrt{3}}{2}\right)$

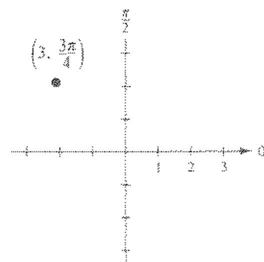
$y = r\sin\theta = -1\left(-\frac{1}{2}\right)$

$(x, y) = \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$



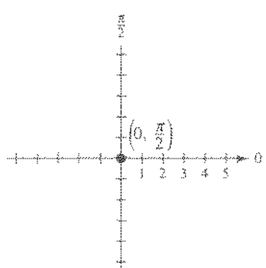
109. $(r, \theta) = \left(3, \frac{3\pi}{4}\right)$

$(x, y) = \left(3\cos\frac{3\pi}{4}, 3\sin\frac{3\pi}{4}\right) = \left(\frac{-3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}\right)$



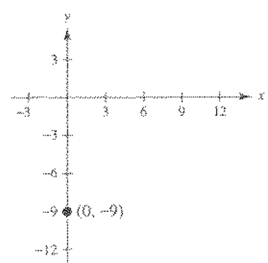
110. $(r, \theta) = \left(0, \frac{\pi}{2}\right)$, the origin

$(x, y) = (0, 0)$



111. $(x, y) = (0, -9)$

$(r, \theta) = \left(9, \frac{3\pi}{2}\right), \left(-9, \frac{\pi}{2}\right)$



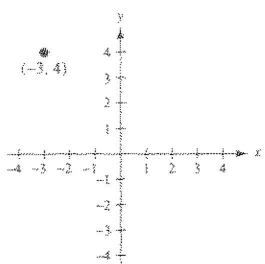
112. $(x, y) = (-3, 4)$

$r = 5$, $\tan\theta = \frac{-4}{3}$

$(5, 126.87°), (-5, 306.87°)$

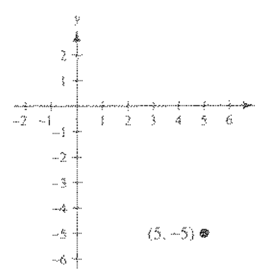
or

$(5, 2.214), (-5, 5.356)$
(radians)



113. $(x, y) = (5, -5)$

$(r, \theta) = \left(5\sqrt{2}, \frac{7\pi}{4}\right), \left(-5\sqrt{2}, \frac{3\pi}{4}\right)$

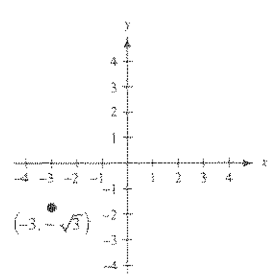


114. $(x, y) = \left(-3, -\sqrt{3}\right)$

Third quadrant, $\theta = \frac{7\pi}{6}$

$r^2 = (-3)^2 + 3 = 12 \implies r = 2\sqrt{3}$

$(r, \theta) = \left(2\sqrt{3}, \frac{7\pi}{6}\right), \left(-2\sqrt{3}, \frac{\pi}{6}\right)$



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115. $x^2 + y^2 = 9$

$r^2 = 9$

$r = 3$

116. $x^2 + y^2 = 20$

$r^2 = 20$

$r = 2\sqrt{5}$

117. $x^2 + y^2 - 4x = 0$

$r^2 - 4r\cos\theta = 0$

$r = 4\cos\theta$

118. $x^2 + y^2 = 6y$

$r^2 = 6r\sin\theta$

$r = 6\sin\theta$

119.
$$xy = 5$$
$$(r\cos\theta)(r\sin\theta) = 5$$
$$r^2 = 5\csc\theta \cdot \sec\theta$$

120.
$$xy = -2$$
$$(r\cos\theta)(r\sin\theta) = -2$$
$$r^2 = -2\sec\theta\csc\theta$$

121.
$$4x^2 + y = 1$$
$$4(r\cos\theta)^2 + (r\sin\theta)^2 = 1$$
$$4r^2\cos^2\theta + r^2(1 - \cos^2\theta) = 1$$
$$r^2[3\cos^2\theta + 1] = 1$$
$$r^2 = \frac{1}{3\cos^2\theta + 1}$$

122.
$$2x^2 + 3y^2 = 1$$
$$2(r\cos\theta)^2 + 3(r\sin\theta)^2 = 1$$
$$r^2(2\cos^2\theta + 3\sin^2\theta) = 1$$
$$r^2(2(1 - \sin^2\theta) + 3\sin^2\theta) = 1$$
$$r^2(2 + \sin^2\theta) = 1$$
$$r^2 = \frac{1}{2 + \sin^2\theta}$$

123. $r = 5$

$x^2 + y^2 = 5^2 = 25$

Circle

124. $r = 12$

$x^2 + y^2 = 144$

Circle

125. $r = 3\cos\theta$

$r^2 = 3r\cos\theta$

$x^2 + y^2 = 3x$

126. $r = 8\sin\theta$

$r^2 = 8r\sin\theta$

$x^2 + y^2 = 8y$

127.
$$r^2 = \cos 2\theta$$
$$r^2 = 1 - 2\sin^2\theta$$
$$r^4 = r^2 - 2r^2\sin^2\theta$$
$$(x^2 + y^2)^2 = x^2 + y^2 - 2y^2$$
$$(x^2 + y^2)^2 - x^2 + y^2 = 0$$

128.
$$r^2 = \sin\theta$$
$$r^3 = r\sin\theta$$
$$(x^2 + y^2)^{3/2} = y \text{ or}$$
$$(x^2 + y^2)^3 = y^2$$

129. $\theta = \dfrac{5\pi}{6}$

$\tan\theta = \dfrac{y}{x} = -\dfrac{1}{\sqrt{3}}$

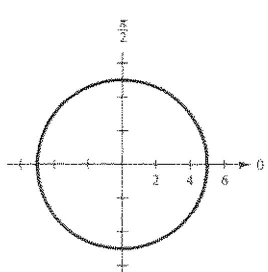
$y = -\dfrac{\sqrt{3}}{3}x,$ line

130. $\theta = \dfrac{4\pi}{3}$

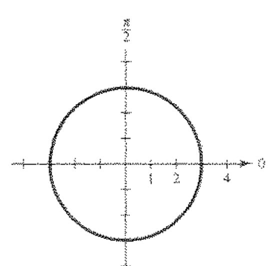
$\tan\theta = \dfrac{y}{x} = \sqrt{3}$

$y = \sqrt{3}x$

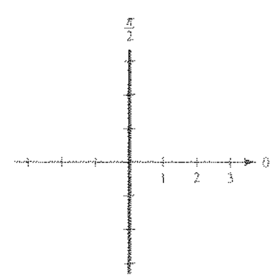
131. $r = 5$, circle



132. $r = 3$, circle



133. $\theta = \dfrac{\pi}{2}$, y-axis



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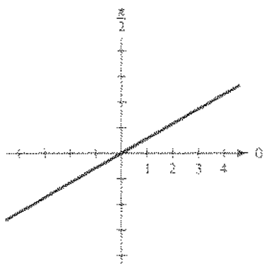
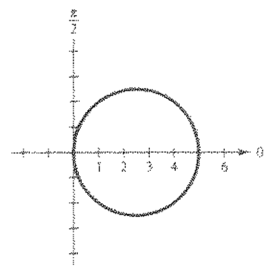
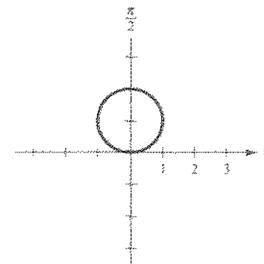
134. $\theta = -\dfrac{5\pi}{6}$, line



135. $r = 5\cos\theta$, circle



136. $r = 2\sin\theta$, circle



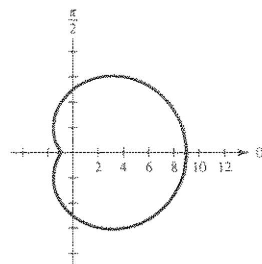
137. $r = 5 + 4\cos\theta$

Dimpled limaçon

Symmetric with respect to polar axis

r is maximum at $\theta = 0$: $(r, \theta) = (9, 0)$

$r \neq 0$ (No zeros)



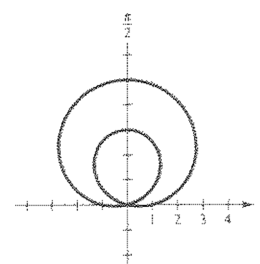
138. $r = 1 + 4\sin\theta$

Limaçon with inner loop

Symmetric with respect to $\theta = \dfrac{\pi}{2}$

$|r|$ is a maximum at $\theta = \dfrac{\pi}{2}$; $\left(5, \dfrac{\pi}{2}\right)$

$r = 0$ when $4\sin\theta = -1 \implies \sin\theta = -\dfrac{1}{4} \implies \theta \approx 3.394, 6.031$



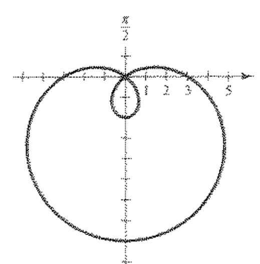
139. $r = 3 - 5\sin\theta$

Limaçon with loop

Symmetry: line $\theta = \dfrac{\pi}{2}$

Maximum $|r|$-value: $|r| = 8$ when $\theta = \dfrac{3\pi}{2}$

Zeros: $r = 0$ when $\theta \approx 0.6435, 2.4981$ $\left(\sin\theta = \dfrac{3}{5}\right)$



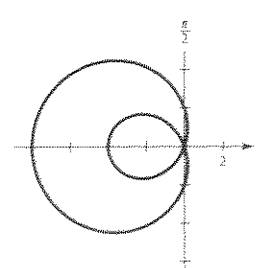
140. $r = 2 - 6\cos\theta$

Limaçon with inner loop

Symmetry: polar axis

Maximum: $|r| = 8$ when $\theta = \pi$

Zero: $r = 0$ when $\cos\theta = \frac{1}{3} \implies \theta \approx 1.231, 5.052$



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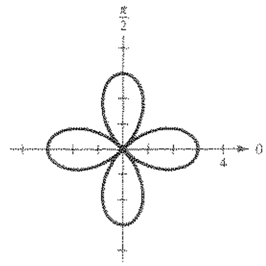
141. $r = -3\cos 2\theta,\ 0 \le \theta \le 2\pi$

Four-leaved rose

Symmetric with respect to $\theta = \dfrac{\pi}{2}$, polar axis, and pole

The value of $|r|$ is a maximum (3) at $\theta = 0, \dfrac{\pi}{2}, \pi, \dfrac{3\pi}{2}$.

$r = 0$ for $\theta = \dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{5\pi}{4}, \dfrac{7\pi}{4}$



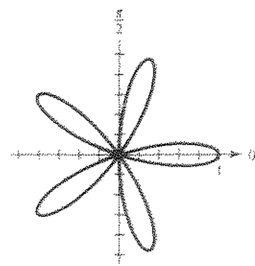
142. $r = \cos 5\theta$

Five-leaved rose

Symmetric with respect to polar axis

$|r|$ is maximum value of 1 at $\theta = \dfrac{n\pi}{5}, n = 0, 1, 2, \ldots$

$r = 0$ for $\theta = \dfrac{\pi}{10} + \dfrac{2n\pi}{10}, n = 0, 1, 2, \ldots$



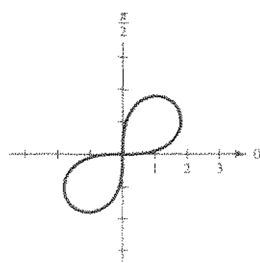
143. $r^2 = 5\sin 2\theta$

Lemniscate

Symmetry with respect to pole

Maximum $|r|$-value: $\sqrt{5}$ when $\theta = \dfrac{\pi}{4}, \dfrac{5\pi}{4}$

Zeros: $r = 0$ when $\theta = 0, \dfrac{\pi}{2}, \pi, \dfrac{3\pi}{2}$



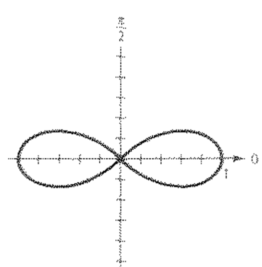
144. $r^2 = \cos 2\theta$

Lemniscate

Symmetry: Pole, polar axis, and line $\theta = \dfrac{\pi}{2}$

Maximum: $|r| = 1$ when $\theta = 0, \pi, 2\pi$

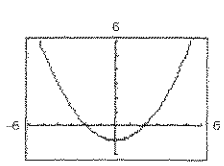
Zeros: $r = 0$ when $\theta = \dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{5\pi}{4}, \dfrac{7\pi}{4}$



145. $r = \dfrac{2}{1 - \sin\theta}$

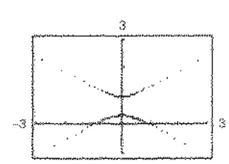
$e = 1$

Parabola



146. $r = \dfrac{1}{1 + 2\sin\theta}, e = 2$

Hyperbola symmetric with $\theta = \pi/2$ and having vertices at $(1/3, \pi/2)$ and $(-1, 3\pi/2)$



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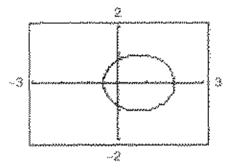
147. $r = \dfrac{4}{5 - 3\cos\theta}$

$= \dfrac{4/5}{1 - (3/5)\cos\theta}$

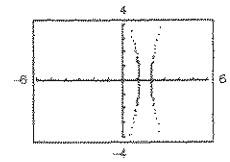
$e = \dfrac{3}{5}$

Ellipse



148. $r = \dfrac{6}{-1 + 4\cos\theta} = \dfrac{-6}{1 - 4\cos\theta}$

Hyperbola ($e = 4$)

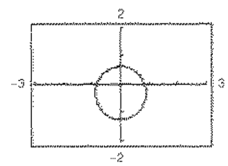


149. $r = \dfrac{5}{6 + 2\sin\theta}$

$= \dfrac{5/6}{1 + (1/3)\sin\theta}$

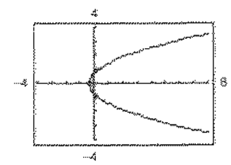
$e = \dfrac{1}{3}$

Ellipse



150. $r = \dfrac{3}{4 - 4\cos\theta} = \dfrac{3/4}{1 - \cos\theta}$

Parabola ($e = 1$)



151. $e = 1$

$r = \dfrac{4}{1 - \cos\theta}$

Vertical directrix: $x = -4$

152. Parabola: $r = \dfrac{ep}{1 + e\sin\theta}$, $e = 1$

Vertex: $\left(2, \dfrac{\pi}{2}\right)$

Focus: $(0, 0) \implies p = 4$

$r = \dfrac{4}{1 + \sin\theta}$

153. Ellipse: $r = \dfrac{ep}{1 - e\cos\theta}$

Vertices: $(5, 0), (1, \pi) \implies a = 3$

One focus: $(0, 0) \implies c = 2$

$e = \dfrac{c}{a} = \dfrac{2}{3}, 5 = \dfrac{2/3\,p}{1 - (2/3)\cos 0} \implies p = \dfrac{5}{2}$

$r = \dfrac{(2/3)(5/2)}{1 - (2/3)\cos\theta} = \dfrac{5/3}{1 - (2/3)\cos\theta} = \dfrac{5}{3 - 2\cos\theta}$

154. Hyperbola: $r = \dfrac{ep}{1 + e\cos\theta}$

Vertices: $(1, 0), (7, 0) \implies a = 3$

One focus: $(0, 0) \implies c = 4$

$e = \dfrac{c}{a} = \dfrac{4}{3}, 1 = \dfrac{4/3\,p}{1 + (4/3)\cos 0} \implies p = \dfrac{7}{4}$

$r = \dfrac{(4/3)(7/4)}{1 + (4/3)\cos\theta} = \dfrac{7/3}{1 + (4/3)\cos\theta} = \dfrac{7}{3 + 4\cos\theta}$

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155. $e = 0.093$

Use $r = \dfrac{ep}{1 - e\cos\theta}$.

$$2a = \frac{0.093p}{1 - 0.093\cos 0} + \frac{0.093p}{1 - 0.093\cos\pi} = 0.1876p = 3.05 \implies p \approx 16.258,\ ep \approx 1.512$$

$$r = \frac{1.512}{1 - 0.093\cos\theta}$$

Perihelion: $\dfrac{1.512}{1 + 0.093} \approx 1.383$ astronomical units

Aphelion: $\dfrac{1.512}{1 - 0.093} \approx 1.667$ astronomical units

156. Use $r = \dfrac{ep}{1 - e\sin\theta}$ (horizontal directrix below pole).

$e = 1$ (parabola)

When $\theta = \dfrac{-\pi}{2}$, $r = 6{,}000{,}000$.

$$r = \frac{p}{1 - \sin\left(\dfrac{-\pi}{2}\right)} = \frac{p}{2} = 6{,}000{,}000 \implies p = 12{,}000{,}000$$

$$r = \frac{12{,}000{,}000}{1 - \sin\theta}$$

When $\theta = -\dfrac{\pi}{3}$, distance is approximately 6,430,781 miles.

157. False. The y^4-term is not second degree.

158. False. There are many sets possible. For example,

$x = t,\ y = 3 - 2t$

$x = 3t,\ y = 3 - 6t$.

159. (a) Vertical translation

(b) Horizontal translation

(c) Reflection in the y-axis

(d) Parabola opens more slowly.

160. (a) Major axis horizontal

(b) Circle

(c) Ellipse is flatter.

(d) Horizontal translation

161. The number b must be less than 5. The ellipse becomes more circular and approaches a circle of radius 5.

162. The orientation of the graph would be reversed.

163. (a) The speed would double.

(b) The elliptical orbit would be flatter. The length of the major axis is greater.

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Chapter 9 Practice Test

1. Find the vertex, focus and directrix of the parabola $x^2 - 6x - 4y + 1 = 0$.

2. Find an equation of the parabola with its vertex at $(2, -5)$ and focus at $(2, -6)$.

3. Find the center, foci, vertices, and eccentricity of the ellipse $x^2 + 4y^2 - 2x + 32y + 61 = 0$.

4. Find an equation of the ellipse with vertices $(0, \pm 6)$ and eccentricity $e = \frac{1}{2}$.

5. Find the center, vertices, foci, and asymptotes of the hyperbola $16y^2 - x^2 - 6x - 128y + 231 = 0$.

6. Find an equation of the hyperbola with vertices at $(\pm 3, 2)$ and foci at $(\pm 5, 2)$.

7. Rotate the axes to eliminate the xy-term. Sketch the graph of the resulting equation, showing both sets of axes.
   $5x^2 + 2xy + 5y^2 - 10 = 0$

8. Use the discriminant to determine whether the graph of the equation is a parabola, ellipse, or hyperbola.
   (a) $6x^2 - 2xy + y^2 = 0$
   (b) $x^2 + 4xy + 4y^2 - x - y + 17 = 0$

For Exercises 9 and 10, eliminate the parameter and write the corresponding rectangular equation.

9. $x = 3 - 2\sin\theta,\ y = 1 + 5\cos\theta$

10. $x = e^{2t},\ y = e^{4t}$

11. Convert the polar point $\left(\sqrt{2}, (3\pi)/4\right)$ to rectangular coordinates.

12. Convert the rectangular point $\left(\sqrt{3}, -1\right)$ to polar coordinates.

13. Convert the rectangular equation $4x - 3y = 12$ to polar form.

14. Convert the polar equation $r = 5\cos\theta$ to rectangular form.

15. Sketch the graph of $r = 1 - \cos\theta$.

16. Sketch the graph of $r = 5\sin 2\theta$.

17. Sketch the graph of $r = \dfrac{3}{6 - \cos\theta}$.

18. Find a polar equation of the parabola with its vertex at $(6, \pi/2)$ and focus at $(0, 0)$.

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